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WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS

BY
W. M. WILSON
AND
G. A. MANEY

BULLETIN No. 80

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ENGINEERING EXPERIMENT STATION

BULLETIN No. 80 JUNE, 1915

WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS.

BY W. M. WILSON, ASSISTANT PROFESSOR OF STRUCTURAL ENGINEERING, AND G. A. MANEY, FORMERLY RESEARCH FELLOW IN DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS.*

CONTENTS.

*While the bulletin is the result of the joint efforts of the two writers, certain parts are so distinctly the work of one that it seems desirable to make a statement relative to the part which should be credited to each. The method of making the analysis involving the use of the slope-deflection equations should be credited to G. A. Maney. The conception of the bulletin as a whole, the method of presenting the results, and the authorship of the text should be credited to W. M. Wilson. The numerical calculations of Tables 11 to 22, inclusive, were made by both writers.—The Editor.

LIST OF TABLES.

LIST OF FIGURES.

WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS

I. Introduction.

1. *Preliminary.*—The increase in the price of land in large cities has made it necessary to build high buildings in order to get a large rentable floor space on a small parcel of land. The type of building generally used is known as the steel-skeleton building. In this type of building the live and dead loads, including the weight of the walls, are carried by a system of beams and girders to columns and are carried by the columns to the footings.

In high buildings the horizontal shear due to the wind load is very large; and, since it is usually impracticable to put diagonal braces between the columns, it is customary to make the steel frame rigid enough to resist the horizontal shear by virtue of the stiffness of the columns and girders. The exact determination of the stresses in a steel frame due to a horizontal shear is one of the problems of structural engineering which remains to be solved. While the writers realize that the method of determining these stresses presented in this bulletin is based upon assumptions which are not exactly true, they believe that the method is more accurate than the methods ordinarily used.

2. *Acknowledgment.*—Messrs. Anderson, Becker, Gomez, and Richart, graduate students in the College of Engineering of the University of Illinois, calculated the moments in Table 23, and the moments as determined by methods I, II, and III in the keys to the diagrams in Figs. 10, 11, and 12, and assisted with the calculations necessary to determine the curves shown in these figures. Professor Ira O. Baker and Professor C. A. Ellis rendered valuable assistance, criticizing the bulletin during its prepartion. The writers gratefully acknowledge indebtedness to these men.

II. Present Methods of Calculating Wind Stresses in Office Buildings.

3. *Classification of Methods.*—Methods of calculating wind stresses in the steel frames of office buildings may be divided into two classes, viz.: (1), those used in the actual design of buildings and (2), those

which have resulted from an effort to make an exact analysis of the stresses. In the methods of the first class, accuracy has been sacrificed to shorten and simplify the calculations. In the methods of the second class, the aim has been to make an exact analysis rather than an analysis that can be used in the actual design of a building.

For the sake of convenience in reference, methods of the first class are designated as *approximate methods,* and those of the second class are designated as *exact methods.*

4. *Approximate Methods.*—(a) Fleming's Methods. Mr. R. Fleming, in an article in *Engineering News,* describes three methods, which are in current use.* These methods are designated as methods I, II, and III. The three methods, as applied to a building in which all columns of a story have the same section, are based upon the following assumptions:

Assumptions in Method I.

1. A bent of a frame acts as a cantilever.

2. The point of contra-flexure of each column is at mid-height of the story.

3. The point of contra-flexure of each girder is at its mid-length.

4. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent.

Assumptions in Method II.

1. A bent of a frame acts as a series of portals.

2. The point of contra-flexure of each column is at mid-height of the story.

3. The shear is the same on all columns of a story.

4. Each pair of adjacent columns of a bent acts as a portal, and each interior column is a member of two adjacent portals. The direct stress in an interior column, when the column is considered as a member of the portal on one side, is of opposite sign from the direct stress in the same column when considered as a member of the portal on the opposite side and the resultant direct stress is equal to zero.

*Wind Bracing without Diagonals for Steel-Frame Office Buildings, Engineering News, March 13, 1913.

Assumptions in Method III.

1. A bent of a frame acts as a continuous portal.

2. The point of contra-flexure of each column is at mid-height of the story.

3. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent.

4. The shear is the same on all columns of a story.

(b) Smith's Methods. Professor Albert Smith, in a paper before the Western Society of Engineers, describes a method which he has used in his classes in Structural Engineering at Purdue University.‡ This method is here designated as Method IV.

Assumptions in Method IV.

1. The point of contra-flexure of each column is at mid-height of the story.

2. The point of contra-flexure of each girder is at its mid-length.

3. The shears on the internal columns are equal and the shear on each external column is equal to one-half of the shear on an interior column.

If all of the assumptions of any of these methods are accepted, the stresses in a frame may be determined by applying the fundamental equations of static equilibrium. It is apparent from the assumptions that the results obtained by these methods are radically different.

5. *Exact Methods.*—(a) Melick's Method. Dr. Cyrus A. Melick* used a method which takes into account the form of the elastic curves and the deflections and changes in length of the members. The method is so long that, when applied to a building only four stories high, the amount of work required is almost prohibitive. To apply it to a building twenty stories high would be impracticable if not, in fact, impossible.

(b) Jonson's Method. Mr. Ernest F. Jonson suggested a method which takes into account the deflections of the columns and the changes in the slopes of the tangents to the elastic curves of the columns and girders at the points where they intersect.†

If the method which he suggests were used, it would give the stresses with a fair degree of accuracy; but his method involves so many unknowns that its use would not be practicable in the actual design of buildings.

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‡Wind Stresses in the Frames of Office Buildings, by Albert Smith, Journal Western Society of Engineers, Vol. XX, No. 4, p. 341.

*Stresses in Tall Buildings, by Cyrus A. Melick. Bulletin No. 8, College of Engineering, University of Ohio.

†The Theory of Frameworks with Rectangular Panels and Its Application to Buildings which Have to Resist Wind, by Ernst F. Jonson, Tran. Am. Soc. C. E., Vol. 55, p. 413.

(c) Method of Least Work. Professor Albert Smith has determined the wind stresses in symmetrical bents having two, three, and four spans by the method of least work.* This method is exact, but, of course, is extremely long.

III. Outline of the Proposed Analysis.

6. *Outline of the Method.*—In making an analysis of the stresses, the writers made certain assumptions and applied certain fundamental principles of mechanics and obtained equations from which the stresses in a frame can be determined. The assumptions which have been made are stated in Section IV and the derivation of the equations is given in Sections V and VI.

It can be proven that the moment at an end of a member of a frame is a function of the changes in the slopes of the tangents to the elastic curve of the member at its ends and of the deflection of one end of the member relative to the other end (see equation A, page 13).

In the strained position, all the columns and girders which intersect at one point have been subjected to the same change in slope (see assumption 1, Section IV); the vertical deflections of the ends of all girders are equal to zero; and the horizontal deflections of the tops of all columns of a story are equal (see assumption 2, Section IV).

Consider any story of a bent. Take the point of intersection of the neutral axes of a column and a girder as a free body. It is in equilibrium under the action of the moments at the extremities of the columns and girders which intersect at the point. Each of the moments may be expressed in terms of the changes in the slopes at the extremities of the member, and the deflection of one end of the member relative to the other. A moment equation can therefore be written for each point where the columns and girders intersect, and the only unknown quantities will be the changes in the slopes at the extremities of the columns and the horizontal deflections of the columns in a story.

If all the columns of a story be taken together as a free body, the sum of the moments at the two extremities of all the columns will be balanced by a couple whose moment is equal to the total shear on the story multiplied by the story height. The shear and the height of the story are known, and the moments in the columns can be expressed in terms of the slopes and the deflections at their extremities the same as in the previous equations. It is therefore possible to write as many equations for each story as there are columns in the story, plus one. As the only

*Wind Stresses in the Frames of Office Buildings, by Albert Smith, Journal Western Society of Engineers, Vol. XX, No. 4, p. 341.

unknown quantities in these equations are the changes in the slopes at the extremities of the columns and the deflection in a story common to all columns, there are as many equations per story as there are unknowns. By solving these equations the slopes and the deflections can be determined. Knowing the slopes and the deflections, the moments can be computed.

The product of the shear on a member and the length of the member is equal to the algebraic sum of the moments at the extremities of the member. Since the moments and the length of the member are known the shear can be computed.

With the shears in the girders known, the direct stress in any column can be determined by taking the column as a free body and equating the sum of the vertical forces to zero.

The direct stress in a girder may be determined in a similar manner.

The method just described is based upon the proposition in mechanics used by Mr. Jonson, but the method which the writers have developed differs from the one used by him in that the changes in the slopes and the deflections have been used as the unknown quantities instead of the direct stresses and the moments. Four members, two columns and two girders, intersect in a point. Each member is subjected to a different direct stress and a different moment, whereas all of the members are subjected to the same change in slope, and all of the columns in a story are subjected to the same deflection. It is therefore apparent that there are fewer unknown slopes and deflections than moments and direct stresses. The large reduction in the number of unknowns very much simplifies the solution of the equations.

IV. Assumptions Upon Which the Analysis Is Based.

7. *Statement of Assumptions.*—The proposed analysis is based upon the following assumptions:

1. The connections between the columns and girders are perfectly rigid.

2. The change in the length of a member due to the direct stress is equal to zero.

3. The length of a girder is the distance between the neutral axes of the columns which it connects and the length of a column is the distance between the neutral axes of the girders which it connects.

4. The deflection of a member due to the internal shearing stresses is equal to zero.

5. The wind load is resisted entirely by the steel frame.

These assumptions will be discussed in Section XI.

V. Fundamental Equation.

8. *Fundamental Proposition.*—The fundamental equation used in this analysis is based upon the following proposition:

When a member is subjected to flexure, the deflection of any point in the neutral axis from the tangent to the elastic curve at any other point is equal to the moment of the area of the $\frac{M}{EI}$ *diagram for the portion of the member between the two points, about the point where the deflection is measured.*

9. *Proof of the Proposition.*—The line AB, Fig. 1, represents the neutral axis of a member subjected to flexure. The deflection of the member is very much exaggerated in the figure. The actual deflection is so small that the length of the curve may be considered equal to its horizontal projection. It is required to prove that the deflection of any point P from the tangent to the line AB at any other point Q is equal to $\int_Q^P \frac{M}{EI} dx{\cdot}x$.

Extend the tangents at the extremities of an element of the curve until they intersect the vertical line through P. The intercept on this vertical line between two consecutive tangents is equal to $x{\cdot}d\theta$. The total deflection of P from the tangent at Q is equal to the algebraic sum of these intercepts for the elements of the elastic curve between the points Q and P. That is, $y = \int_Q^P x d\theta$. The equation of the elastic curve may be written $\frac{d\theta}{dx} = \frac{M}{EI}$. Substituting the value of $d\theta$ from this equation in the expression for y, gives $y = \int_Q^P \frac{M}{EI} dx{\cdot}x$. The quantity $\frac{M}{EI} dx$ can be considered as an elementary area of the $\frac{M}{EI}$ diagram. The quantity $\int_Q^P \frac{M}{EI} dx{\cdot}x$ can therefore be considered as the moment of the $\frac{M}{EI}$ diagram about the point P.

10. *Derivation of Fundamental Equation.*—Consider a member which is not acted upon by any external forces or couples except at the ends. The line AB in Fig. 2 represents the neutral axis of such a member. The moment at A is represented by M_{AB} and at B by M_{BA}. The change in the slope of the elastic curve at A due to the external forces is represented by θ_A, and at B by θ_B. The deflection of A from its original position A', is d. The deflection of A from the tangent at B is represented by $(d - l{\cdot}\theta_B)$; and the deflection of B from the tangent

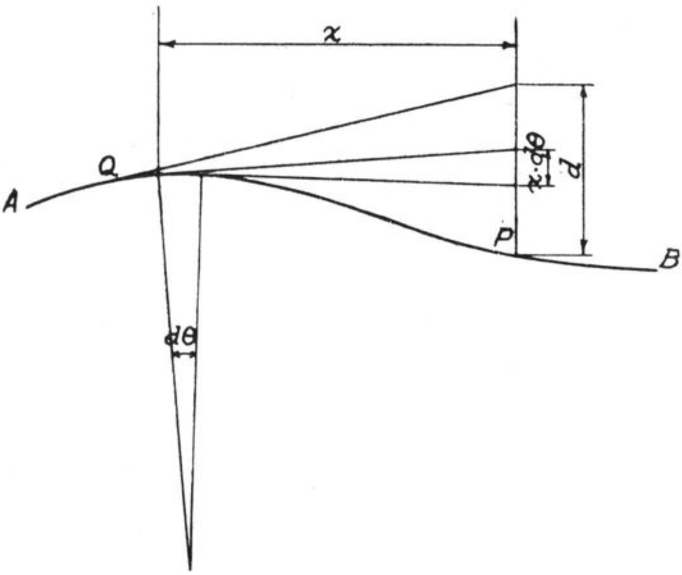

FIG. 1. PROOF OF FUNDAMENTAL EQUATION.

at A is represented by $(d - l{\cdot}\theta_A)$. The $\dfrac{M}{EI}$ diagram is shown in Fig. 3.

The signs of the quantities are determined by the following rules:

The change in the slope, or the angular deformation, is positive (+) when the tangent to the elastic curve of the member is turned in a clockwise direction.

Distances and deflections are positive when they are measured in the same direction from the base line as are positive slopes.

The moment acting upon a member at the section where the deflection is measured is positive (+) when it produces a clockwise rotation.

Substituting $(d - l{\cdot}\theta_B)$, the deflection of A from the tangent at B, for y in the equation $y = \int \frac{M}{EI} dx \cdot x$, gives

$$(d - l \cdot \theta_B) = \int_A^B \frac{M}{EI} dx \cdot x.$$

Substituting the value of M from the equation

$$M = M_{AB} + \left[\frac{M_{BA} - M_{AB}}{l}\right] x, \text{ gives}$$

$$(d - l \cdot \theta_B) = \int_A^B \frac{M_{AB}}{EI} x dx + \left[\frac{M_{BA} - M_{AB}}{EI \cdot l}\right] x^2 \cdot dx.$$

If the material is homogeneous and the section uniform, E and I are constants. Performing the indicated integration, gives

$$(d - l \cdot \theta_B) = \frac{M_{AB}}{EI} \frac{l^2}{2} + \frac{M_{BA}}{EI} \frac{l^2}{3} - \frac{M_{AB}}{EI} \frac{l^2}{3}, \text{ or}$$

$$d = l \cdot \theta_B + \frac{l^2}{6EI} \left[2M_{BA} + M_{AB} \right] \quad \ldots\ldots\ldots\ldots (1)$$

Substituting $(d - l \cdot \theta_A)$, the deflection of B from the tangent at A, for y in the equation, $y = \int \frac{M}{EI} dx \cdot x$, gives

$$(d - l \cdot \theta_A) = \int_B^A \frac{M}{EI} dx \cdot x.$$

The moment at any section in the member, when considering the deflection at B, is of opposite sign from the moment at the same section when considering the deflection at A. (See preceding rule for determining the sign of the moment.)

Substituting the value of M from the equation

$$M = - M_{BA} - \left[\frac{M_{AB} - M_{BA}}{l}\right] x, \text{ gives}$$

$$(d - l \cdot \theta_A) = \int - \frac{M_{BA}}{EI} x dx - \left[\frac{M_{AB} - M_{BA}}{l}\right] x^2 \, dx \text{ or}$$

$$d = l \cdot \theta_A - \frac{M_{BA}}{EI} \frac{l^2}{2} - \frac{M_{AB}}{EI} \frac{l^2}{3} + \frac{M_{BA}}{EI} \frac{l^2}{3}$$

$$d = l \cdot \theta_A + \frac{l^2}{6EI} \left[- M_{BA} - 2M_{AB} \right] \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Multiplying equation (2) by 2, gives

$$2d = 2l \cdot \theta_A + \frac{l^2}{6EI} \left[- 2M_{BA} - 4M_{AB} \right] \quad \ldots\ldots\ldots\ldots (3)$$

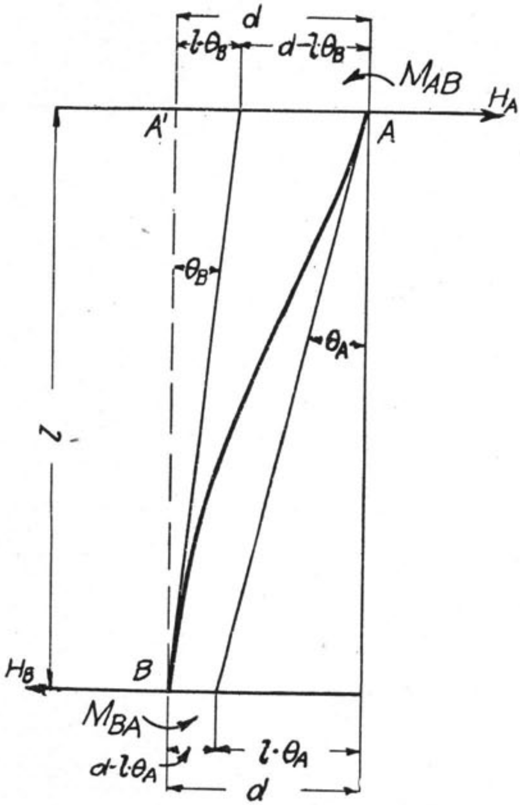

FIG. 2. DERIVATION OF FUNDAMENTAL EQUATION.

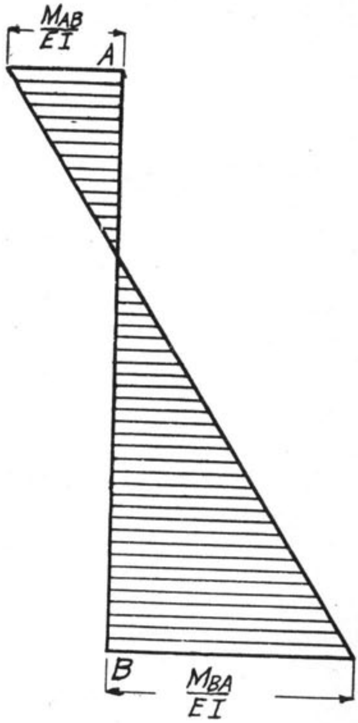

FIG. 3. $\dfrac{M}{EI}$ DIAGRAM.

Adding equations (1) and (3) gives

$$3d = 2l{\cdot}\theta_A + l{\cdot}\theta_B + \frac{l^2}{6EI}\Big[-3M_{AB}\Big].$$

Substituting K for $\dfrac{I}{l}$ and R for $\dfrac{d}{l}$ and solving for M_{AB}, gives,

$$M_{AB} = 2\,EK\ \ [2\,\theta_A + \theta_B - 3R] \ldots\ldots\ldots\ldots\ldots \text{(A)}$$

When $d = 0$, equation (A) becomes

$$M_{AB} = 2\,EK\ \ (2\,\theta_A + \theta_B) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(B)}$$

Equation (A) is general and may be applied to any length of any member in bending provided the length considered has no intermediate external force applied to it. That is, one or more of the quantities

θ_A, θ_B, and d may be negative and equation (A) will still give the moment at the point A in both magnitude and sign. Equation (A) is the fundamental equation upon which the analysis which follows is based. Equation (B) is a special form of equation (A). Equation (A) may be expressed as follows:

The moment at the end of any member is equal to 2EK times the quantity: Two times the change in the slope at the near end plus the change in the slope at the far end minus three times the deflection divided by the length. E is the modulus of elasticity of the material and K is the ratio of the moment of inertia to the length of the member.

VI. Derivation of General Equations.

11. *Notation.*—The following notation has been used:

A, B, C, etc. = the columns of a bent, beginning at the right and reading toward the left.

a, b, c, etc.= the bays of a bent, beginning at the right and reading toward the left. The girder in bay a is designated as girder a, in bay b as girder b, in bay c as girder c, etc.

A_1, A_2, A_3, etc. = the intersections of the neutral axes of the girders at the tops of the first, second, third, etc., stories with the neutral axis of column A.

B_1, B_2, B_3, etc. = the intersections of the neutral axes of the girders at the tops of the first, second, third, etc., stories with the neutral axis of column B.

d = deflection of the columns in a story height.

E = modulus of elasticity of the material.

h = length of a column measured from neutral axis to neutral axis of the girders.

I = moment of inertia of the girder and column sections.

$J = 2\sum\left(\frac{I}{l}+\frac{I}{h}\right)$ for all columns and girders which intersect at a point.

$K = \frac{I}{l}$ for girders and $\frac{I}{h}$ for columns.

l = length of a girder measured from neutral axis to neutral axis of the columns.

M = bending moment.

$N = 2\sum\left(\frac{I}{h}\right)$ for all columns in a story.

$R = \frac{d}{h}$

W = total horizontal shear in a bent at any story.

w = increment of the horizontal shear in a bent in a story height.

θ = change in the slope of the tangent to the elastic curve.

Subscripts are added to the letters to indicate the particular part of a bent to which a given symbol applies. For example, referring to Fig. 4, girder b_1 is the girder in bay b at the top of the first story, A_1 is the intersection of the girder at the top of the first story with

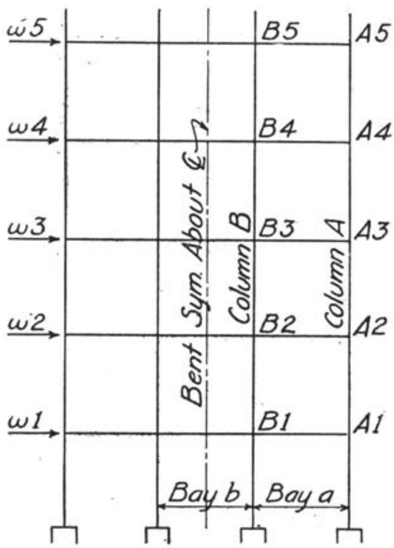

Fig. 4. Bottom Five Stories of a Symmetrical Three-Span Bent.

column A, θ_{A_1} is the slope of the tangent to the elastic curve at the point A_1; d_3 is the deflection of the columns in the third story; J_{B_4} is the J at the point B_4; K_{A_2} is the K of column A in the second story. The moment at the right end of the girder in bay a at the top of the second story is designated as M_2^{AB}, and the moment at the left end of the same girder is designated as M_2^{BA}. The moment at the top of the column B in the third story is designated as M_B^{32}, and the moment at the bottom of the same column is designated as M_B^{23}.

12. *Derivation of Equations.*—Fig. 4 shows the bottom five stories of a symmetrical three-span bent. It is required to find the stresses in all of the members.

Consider the columns of the third story acting together as a free body. (Any other story could have been used.) The algebraic sum of all of the moments at the tops and the bottoms of all the columns plus the product of the total shear in the story and the story height, is equal to zero. That is, $2\,(M_A^{32} + M_A^{23} + M_B^{32} + M_B^{23}) + W_3 h_3 = 0$

Substituting the values of the moments as given by equations A and B of Section V gives

$$2\left[2EK_{A_3}\,(2\theta_{A_3} + \theta_{A_2} - 3R_3) + 2EK_{A_3}\,(2\theta_{A_2} + \theta_{A_3} - 3R_3) + 2EK_{B_3}\,(2\theta_{B_3} + \theta_{B_2} - 3R_3) + 2EK_{B_3}\,(2\theta_{B_2} + \theta_{B_3} - 3R_3)\right] + W_3 h_3 = 0.$$

Letting $N = 2 \sum\left(\frac{I}{h}\right)$ for all of the columns in a story, and collecting, gives

$$2K_{A_3}\,\theta_{A_2} + 2K_{B_3}\,\theta_{B_2} - N_3 R_3 + 2K_{A_3}\,\theta_{A_3} + 2K_{B_3}\,\theta_{B_3} = -\frac{W_3 h_3}{6E} \quad \ldots\ldots (1)$$

Consider the point A_3 as a free body. Taking $\sum M = 0$, gives $M_A^{34} + M_A^{32} + M_3^{AB} = 0$. Substituting the values of these moments as given by equations A and B gives,

$$2EK_{A_4}\,(2\theta_{A_3} + \theta_{A_4} - 3R_4) + 2EK_{A_3}\,(2\theta_{A_3} + \theta_{A_2} - 3R_3) + 2EK_{a_3}\;(2\theta_{A_3} + \theta_{B_3}) = 0.$$

Combining the coefficients of the unknowns and cancelling the common factor $2E$ gives,

$$\theta_{A_2} K_{A_3} + \theta_{A_3}\,(2K_{A_4} + 2K_{A_3} + 2K_{a_3}) + \theta_{A_4}\,K_{A_4} + \theta_{B_3} K_{a_3} - R_3 3K_{A_3} - R_4 3K_{A_4} = 0.$$

Substituting J_{A_3} for $2\sum\left(\frac{I}{h} + \frac{I}{l}\right)$ of all the members that intersect at A_3, the equation becomes:

$$K_{A_3}\,\theta_{A_2} - 3K_{A_3}\,R_3 + J_{A_3}\,\theta_{A_3} + K_{a_3}\,\theta_{B_3} - 3K_{A_4} R_4 + K_{A_4}\theta_{A_4} = 0 \ldots (2)$$

The point B_3 is in equilibrium under the action of the four moments M_3^{BA}, M_3^{BB}, M_B^{34}, and M_B^{32}. Equating the sum of these four moments to zero gives,

$$2EK_{a_3}\,(2\theta_{B_3} + \theta_{A_3}) + 2EK_{b_3}\,(2\theta_{B_3} + \theta_{B_3}) + 2EK_{B_4}\,(2\theta_{B_3} + \theta_{B_4} - 3R_4) + 2EK_{B_3}\,(2\theta_{B_3} + \theta_{B_2} - 3R_3) = 0.$$

Combining the coefficients for the unknowns and cancelling the common factor $2E$, gives,

$$2\theta_{B_3}\,(K_{a_3} + K_{b_3} + K_{B_4} + K_{B_3}) + \theta_{A_3} K_{a_3} + \theta_{B_3} K_{b_3} + \theta_{B_4} K_{B_4} + \theta_{B_2} K_{B_3} - 3R_4 K_{B_4} - 3R_3 K_{B_3} = 0.$$

Substituting J_{B_3} for $2\sum\left(\frac{I}{h} + \frac{I}{l}\right)$ of all the members intersecting at B_3, the equation becomes,

$K_{B_3}\theta_{B_2} - 3K_{B_3}R_3 + K_{a_3}\theta_{A_3} + (K_{b_3} + J_{B_3})\theta_{B_3} - 3K_{B_4}R_4 +$
$K_{B_4}\theta_{B_4} = 0$ ....(3)

The three equations, 1, 2, and 3, can be written for any story by making the proper changes in the subscripts. As there are only three unknowns, θ_A, θ_B, and R for each story, as many equations can be written for a bent as there are unknowns to be determined. It is possible to solve these equations for the unknown quantities algebraically, but the large number of equations involved makes the work very difficult. It is simpler to substitute the numerical values of the coefficients in the equations and solve for the numerical values of the unknown quantities by the process of elimination explained in Section VII.

For convenience in the application of this process of elimination, the equations are written in tabular form as shown in Table 3 in which the unknown changes in the slopes and the ratios of deflection to story height are written at the tops of the columns and the coefficients of these unknowns are written below. For example, in writing equation A of Table 3, which is $-N_1R_1 + 2K_{A_1}\theta_{A_1} + 2K_{B_1}\theta_{B_1} = -\frac{W_1h_1}{6E}$ $-N_1$, the coefficient of R_1, is placed in the column under R_1; $2K_{A_1}$, the coefficient of θ_{A_1}, is placed in the column under θ_{A_1}; $2K_{B_1}$, the coefficient of θ_{B_1}, is placed in the column under θ_{B_1}; and $-\frac{W_1h_1}{6E}$ is placed in the column headed "Right-Hand Member of Equation." Having the equations written in this form, it is unnecessary to repeat the unknown quantities when eliminating them from the equations by the method used in Table 14.

Table 3 contains the general equations to be used in determining the slopes and the deflections in a symmetrical three-span bent any number of stories high. The subscripts 1, 2, and 3 refer to the first, second, and third stories respectively, and the subscripts x, y, and z refer to the second from the top, the next to the top, and the top stories respectively. The equations for the intervening stories are of the same form, but have different subscripts.

By using the general method outlined above, that is, by writing an equation similar to equation 1 for each story and an equation similar to equations 2 and 3 for each intersection of a column with a girder for each story, as many equations as there are unknowns can be written for a bent containing any number of spans and any number of stories. General equations for *symmetrical* bents of from one to five spans are given in Tables 1 to 5 and similar equations for *unsymmetrical* bents of from one to five spans are given in Tables 6 to 10.

In order to check the equations of Tables 1 to 10, a model of a

TABLE 1.

GENERAL EQUATIONS FOR A SYMMETRICAL SINGLE-SPAN BENT ANY NUMBER OF STORIES HIGH.

Equation	Left-Hand Member of Equation													Right-Hand Member of Equation
	Story No. 1		Story No. 2		Story No. 3		Intervening Stories	Story No. X (Second from Top)		Story No. Y (Next to Top)		Story No. Z (Top)		
	R_1	θ_{A1}	R_2	θ_{A2}	R_3	θ_{A3}		R_X	θ_{AX}	R_Y	θ_{AY}	R_Z	θ_{AZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height													
A	$-N_1$	$2K_{A1}$												$-W_1 h_1 \div 6E$
B	$-3K_{A1}$	$J_{A1}+K_{a1}$	$-3K_{A2}$	K_{A2}										O
C		$2K_{A2}$	$-N_2$	$2K_{A2}$										$-W_2 h_2 \div 6E$
D		K_{A2}	$-3K_{A2}$	$J_{A2}+K_{a2}$	$-3K_{A3}$	K_{A3}								O
	Similar equations for intervening stories													
W									$2K_{AY}$	$-N_Y$	$2K_{AY}$			$-W_Y h_Y \div 6E$
X									K_{AY}	$-3K_{AY}$	$J_{AY}+K_{aY}$	$-3K_{AZ}$	K_{AZ}	O
Y											$2K_{AZ}$	$-N_Z$	$2K_{AZ}$	$-W_Z h_Z \div 6E$
Z											K_{AZ}	$-3K_{AZ}$	$J_{AZ}+K_{aZ}$	O

TABLE 2.

GENERAL EQUATIONS FOR A SYMMETRICAL TWO-SPAN BENT ANY NUMBER OF STORIES HIGH.

Equation	Left-Hand Member of Equation																			Right-Hand Member of Equation
	Story No. 1			Story No 2			Story No. 3			Intervening Stories	Story No. X (Second from Top)			Story No. Y (Next to Top)			Story No. Z (Top)			
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	θ_{A3}	θ_{B3}		R_X	θ_{AX}	θ_{AY}	R_Y	θ_{AY}	θ_{BY}	R_Z	θ_{AZ}	θ_{BZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																			
A	$-N_1$	$2K_{A1}$	K_{B1}																	$-W_1h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}	$-3K_{A2}$	K_{A2}															O
C	$-3K_{B1}$	$2K_{a1}$	J_{B1}	$-3K_{B2}$		K_{B2}														O
D		$2K_{A2}$	K_{B2}	$-N_2$	$2K_{A2}$	K_{B2}														$-W_2h_2 \div 6E$
E		K_{A2}		$-3K_{A2}$	J_{A2}	K_{a2}	$-3K_{A3}$	K_{A3}												O
F			K_{B2}	$-3K_{B2}$	$2K_{a2}$	J_{B2}	$-3K_{B3}$		K_{B3}											O
	Similar equations for intervening stories																			
U												$2K_{AY}$	K_{BY}	$-N_Y$	$2K_{AY}$	K_{BY}				$-W_Yh_Y \div 6E$
V												K_{AY}		$-3K_{AY}$	J_{AY}	K_{aY}	$-3K_{AZ}$	K_{AZ}		O
W													K_{BY}	$-3K_{BY}$	$2K_{aY}$	J_{BY}	$-3K_{BZ}$		K_{BZ}	O
X															$2K_{AZ}$	K_{BZ}	$-N_Z$	$2K_{AZ}$	K_{BZ}	$-W_Zh_Z \div 6E$
Y															K_{AZ}		$-3K_{AZ}$	J_{AZ}	K_{aZ}	O
Z																K_{BZ}	$-3K_{BZ}$	$2K_{aZ}$	J_{BZ}	O

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Table 3.

General Equations for a Symmetrical Three-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation																			Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3			Intervening Stories	Story No. X (Second from Top)			Story No. Y (Next to Top)			Story No. Z (Top)			
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	θ_{A3}	θ_{B3}		R_X	θ_{AX}	θ_{BX}	R_Y	θ_{AY}	θ_{BY}	R_Z	θ_{AZ}	θ_{BZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																			
A	$-N_1$	$2K_{A1}$	$2K_{B1}$																	$-W_1 h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}	$-3K_{A2}$	K_{A2}															O
C	$-3K_{B1}$	K_{a1}	$J_{B1}+K_{b1}$	$-3K_{B2}$		K_{B2}														O
D		$2K_{A2}$	$2K_{B2}$	$-N_2$	$2K_{A2}$	$2K_{B2}$														$-W_2 h_2 \div 6E$
E		K_{A2}		$-3K_{A2}$	J_{A2}	K_{a2}	$-3K_{A3}$	K_{A3}												O
F			K_{B2}	$-3K_{B2}$	K_{a2}	$J_{B2}+K_{b2}$	$-3K_{B3}$		K_{B3}											O
	Similar equations for intervening stories																			
U												$2K_{AY}$	$2K_{BY}$	$-N_Y$	$2K_{AY}$	$2K_{BY}$				$-W_Y h_Y \div 6E$
V												K_{AY}		$-3K_{AY}$	J_{AY}	K_{aY}	$-3K_{AZ}$	K_{AZ}		O
W													K_{BY}	$-3K_{BY}$	K_{aY}	$J_{BY}+K_{bY}$	$-3K_{BZ}$		K_{BZ}	O
X															$2K_{AZ}$	$2K_{BZ}$	$-N_Z$	$2K_{AZ}$	$2K_{BZ}$	$-W_Z h_Z \div 6E$
Y															K_{AZ}		$-3K_{AZ}$	J_{AZ}	K_{aZ}	O
Z																K_{BZ}	$-3K_{BZ}$	K_{aZ}	$J_{BZ}+K_{bZ}$	O

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Table 4.

General Equations for a Symmetrical Four-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation: Story No. 1				Story No. 2				Story No. 3				Intervening Stories	Story No. X (Second from Top)				Story No. Y (Next to Top)				Story No. Z (Top)				Right-Hand Member of Equation
	R_1	θ_{A1}	θ_{B1}	θ_{C1}	R_2	θ_{A2}	θ_{B2}	θ_{C2}	R_3	θ_{A3}	θ_{B3}	θ_{C3}		R_X	θ_{AX}	θ_{BX}	θ_{CX}	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																									
A	$-N_1$	$2K_{A1}$	$2K_{B1}$	K_{C1}																						$-W_1 h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}		$-3K_{A2}$	K_{A2}																				O
C	$-3K_{B1}$	K_{a1}	J_{B1}	K_{b1}	$-3K_{B2}$		K_{B2}																			O
D	$-3K_{C1}$		$2K_{b1}$	J_{C1}	$-3K_{C2}$			K_{C2}																		O
E		$2K_{A2}$	$2K_{B2}$	K_{C2}	$-N_2$	$2K_{A2}$	$2K_{B2}$	K_{C2}																		$-W_2 h_2 \div 6E$
F		K_{A2}			$-3K_{A2}$	J_{A2}	K_{a2}		$-3K_{A3}$	K_{A3}																O
G			K_{B2}		$-3K_{B2}$	K_{a2}	J_{B2}	K_{b2}	$-3K_{B3}$		K_{B3}															O
H				K_{C2}	$-3K_{C2}$		$2K_{b2}$	J_{C2}	$-3K_{C3}$			K_{C3}														O
	Similar equations for intervening stories																									
S															$2K_{AY}$	$2K_{BY}$	K_{CY}	$-N_Y$	$2K_{AY}$	$2K_{BY}$	K_{CY}					$-W_Y h_Y \div 6E$
T															K_{AY}			$-3K_{AY}$	J_{AY}	K_{aY}		$-3K_{AZ}$	K_{AZ}			O
U																K_{BY}		$-3K_{BY}$	K_{aY}	J_{BY}	K_{bY}	$-3K_{BZ}$		K_{BZ}		O
V																	K_{CY}	$-3K_{CY}$		$2K_{bY}$	J_{CY}	$-3K_{CZ}$			K_{CZ}	O
W																			$2K_{AZ}$	$2K_{BZ}$	K_{CZ}	$-N_Z$	$2K_{ZV}$	$2K_{BZ}$	K_{CZ}	$-W_Z h_Z \div 6E$
X																			K_{AZ}			$-3K_{AZ}$	J_{AZ}	K_{aZ}		O
Y																				K_{BZ}		$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}	O
Z																					K_{CZ}	$-3K_{CZ}$		$2K_{bZ}$	J_{CZ}	O

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Table 5.

General Equations for a Symmetrical Five-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation																									Right-Hand Member of Equation
	Story No. 1				Story No. 2				Story No. 3				Intervening Stories	Story No. X (Second from Top)				Story No. Y (Next to Top)				Story No. Z (Top)				
	R_1	θ_{A1}	θ_{B1}	θ_{C1}	R_2	θ_{A2}	θ_{B2}	θ_{C2}	R_3	θ_{A3}	θ_{B3}	θ_{C3}		R_X	θ_{AX}	θ_{BX}	θ_{CX}	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																									
A	$-N_1$	$2K_{A1}$	$2K_{B1}$	$2K_{C1}$																						$-W_1 h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}		$-3K_{A2}$	K_{A2}																				O
C	$-3K_{B1}$	K_{a1}	J_{B1}	K_{b1}	$-3K_{B2}$		K_{B2}																			O
D	$-3K_{C1}$		K_{b1}	$J_{C1}+K_{c1}$	$-3K_{C2}$			K_{C2}																		O
E		$2K_{A2}$	$2K_{B2}$	$2K_{C2}$	$-N_2$	$2K_{A2}$	$2K_{B2}$	$2K_{C2}$																		$-W_2 h_2 \div 6E$
F		K_{A2}			$-3K_{A2}$	J_{A2}	K_{a2}		$-3K_{A3}$	K_{A3}																O
G			K_{B2}		$-3K_{B2}$	K_{a2}	J_{B2}	K_{b2}	$-3K_{B3}$		K_{B3}															O
H				K_{C2}	$-3K_{C2}$		K_{b2}	$J_{C2}+K_{c2}$	$-3K_{C3}$			K_{C3}														O
	Similar equations for intervening stories																									
S															$2K_{AY}$	$2K_{BY}$	$2K_{CY}$	$-N_Y$	$2K_{AY}$	$2K_{BY}$	$2K_{CY}$					$-W_Y h_Y \div 6E$
T															K_{AY}			$-3K_{AY}$	J_{AY}	K_{aY}		$-3K_{AZ}$	K_{AZ}			O
U																K_{BY}		$-3K_{BY}$	K_{aY}	J_{BY}	K_{bY}	$-3K_{BZ}$		K_{BZ}		O
V																	K_{CY}	$-3K_{CY}$		K_{bY}	$J_{CY}+K_{cY}$	$-3K_{CZ}$			K_{CZ}	O
W																			$2K_{AZ}$	$2K_{BZ}$	$2K_{CZ}$	$-N_Z$	$2K_{AZ}$	$2K_{BZ}$	$2K_{CZ}$	$-W_Z h_Z \div 6E$
X																			K_{AZ}			$-3K_{AZ}$	J_{AZ}	K_{aZ}		O
Y																				K_{BZ}		$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}	O
Z																					K_{CZ}	$-3K_{CZ}$		K_{bZ}	$J_{CZ}+K_{cZ}$	O

Table 6.

General Equations for an Unsymmetrical Single-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation																			Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3			Intervening Stories	Story No. X (Second from Top)			Story No. Y (Next to Top)			Story No. Z (Top)			
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	θ_{A3}	θ_{B3}		R_X	θ_{AX}	θ_{BX}	R_Y	θ_{AY}	θ_{BY}	R_Z	θ_{AZ}	θ_{BZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																			
A	$-N_1$	K_{A1}	K_{B1}																	$-W_1h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}	$-3K_{A2}$	K_{A2}															O
C	$-3K_{B1}$	K_{a1}	J_{B1}	$-3K_{B2}$		K_{B2}														O
D		K_{A2}	K_{B2}	$-N_2$	K_{A2}	K_{B2}														$-W_2h_2 \div 6E$
E		K_{A2}		$-3K_{A2}$	J_{A2}	K_{a2}	$-3K_{A3}$	K_{A3}												O
F			K_{B2}	$-3K_{B2}$	K_{a2}	J_{B2}	$-3K_{B3}$		K_{B3}											O
	Similar equations for intervening stories																			
U												K_{AY}	K_{BY}	$-N_Y$	K_{AY}	K_{BY}				$-W_Yh_Y \div 6E$
V												K_{AY}		$-3K_{AY}$	J_{AY}	K_{aY}	$-3K_{AZ}$	K_{AZ}		O
W													K_{BY}	$-3K_{BY}$	K_{aY}	J_{BY}	$-3K_{BZ}$		K_{BZ}	O
X															K_{AZ}	K_{BZ}	$-N_Z$	K_{AZ}	K_{BZ}	$-W_Zh_Z \div 6E$
Y															K_{AZ}		$-3K_{AZ}$	J_{AZ}	K_{aZ}	O
Z																K_{BZ}	$-3K_{BZ}$	K_{aZ}	J_{BZ}	O

TABLE 7.

GENERAL EQUATIONS FOR AN UNSYMMETRICAL TWO-SPAN BENT
ANY NUMBER OF STORIES HIGH.

Equation	Left-Hand Member of Equation																									Right-Hand Member of Equation
	Story No. 1				Story No. 2				Story No. 3				Intervening Stories	Story No. X (Second from Top)				Story No. Y (Next to Top)				Story No Z (Top)				
	R_1	θ_{A1}	θ_{B1}	θ_{C1}	R_2	θ_{A2}	θ_{B2}	θ_{C2}	R_3	θ_{A3}	θ_{B3}	θ_{C3}		R_X	θ_{AX}	θ_{BX}	θ_{CX}	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																									
A	$-N_1$	K_{A1}	K_{B1}	K_{C1}																						$-W_1 h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}		$-3K_{A2}$	K_{A2}																				O
C	$-3K_{B1}$	K_{a1}	J_{B1}	K_{b1}	$-3K_{B2}$		K_{B2}																			O
D	$-3K_{C1}$		K_{b1}	J_{C1}	$-3K_{C2}$			K_{C2}																		O
E		K_{A2}	K_{B2}	K_{C2}	$-N_2$	K_{A2}	K_{B2}	K_{C2}																		$-W_2 h_2 \div 6E$
F		K_{A2}			$-3K_{A2}$	J_{A2}	K_{a2}		$-3K_{A3}$	K_{A3}																O
G			K_{B2}		$-3K_{B2}$	K_{a2}	J_{B2}	K_{b2}	$-3K_{B3}$		K_{B3}															O
H				K_{C2}	$-3K_{C2}$		K_{B2}	J_{C2}	$-3K_{C3}$			K_{C3}														O
	Similar equations for intervening stories																									
S															K_{AY}	K_{BY}	K_{CY}	$-N_Y$	K_{AY}	K_{BY}	K_{CY}					$-W_Y h_Y \div 6E$
T															K_{AY}			$-3K_{AY}$	J_{AY}	K_{aY}		$-3K_{AZ}$	K_{AZ}			O
U																K_{BY}		$-3K_{BY}$	K_{aY}	K_{BY}	K_{bY}	$-3K_{BZ}$		K_{BZ}		O
V																	K_{CY}	$-3K_{CY}$		K_{bY}	J_{CY}	$-3K_{CZ}$			K_{CZ}	O
W																			K_{AZ}	K_{BZ}	K_{CZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	$-W_Z h_Z \div 6E$
X																			K_{AZ}			$-3K_{AZ}$	J_{AZ}	K_{aZ}		O
Y																				K_{BZ}		$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}	O
Z																					K_{CZ}	$-3K_{CZ}$		K_{bZ}	J_{CZ}	O

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Table 8.

General Equations for an Unsymmetrical Three-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation																															Right-Hand Member of Equation
	Story No. 1					Story No. 2					Story No. 3					Intervening Stories	Story No. X (Second from Top)					Story No. Y (Next to Top)					Story No. Z (Top)					
	R_1	θ_{A1}	θ_{B1}	θ_{C1}	θ_{D1}	R_2	θ_{A2}	θ_{B2}	θ_{C2}	θ_{D2}	R_3	θ_{A3}	θ_{B3}	θ_{C3}	θ_{D3}		R_X	θ_{AX}	θ_{BX}	θ_{CX}	θ_{DX}	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	θ_{DY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	θ_{DZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																															
A	$-N_1$	K_{A1}	K_{B1}	K_{C1}	K_{D1}																											$-W_1 h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}			$-3K_{A2}$	K_{A2}																									O
C	$-3K_{B1}$	K_{a1}	J_{B1}	K_{b1}		$-3K_{B2}$		K_{B2}																								O
D	$-3K_{C1}$		K_{b1}	J_{C1}	K_{c1}	$-3K_{C2}$			K_{C2}																							O
E	$-3K_{D1}$			K_{c1}	J_{D1}	$-3K_{D2}$				K_{D2}																						O
F		K_{A2}	K_{B2}	K_{C2}	K_{D2}	$-N_2$	K_{A2}	K_{B2}	K_{C2}	K_{D2}																						$-W_2 h_2 \div 6E$
G		K_{A2}				$-3K_{A2}$	J_{A2}	K_{a2}			$-3K_{A3}$	K_{A3}																				O
H			K_{B2}			$-3K_{B2}$	K_{a2}	J_{B2}	K_{b2}		$-3K_{B3}$		K_{B3}																			O
I				K_{C2}		$-3K_{C2}$		K_{b2}	J_{C2}	K_{c2}	$-3K_{C3}$			K_{C3}																		O
J					K_{D2}	$-3K_{D2}$			K_{c2}	J_{D2}	$-3K_{D3}$				K_{D3}																	O
	Similar equations for intervening stories																															
Q																		K_{AY}	K_{BY}	K_{CY}	K_{DY}	$-N_Y$	K_{AY}	K_{BY}	K_{CY}	K_{DY}						$-W_Y h_Y \div 6E$
R																		K_{AY}				$-3K_{AY}$	J_{AY}	K_{aY}			$-3K_{AZ}$	K_{AZ}				O
S																			K_{BY}			$-3K_{BY}$	K_{aY}	J_{BY}	K_{bY}		$-3K_{BZ}$		K_{BZ}			O
T																				K_{CY}		$-3K_{CY}$		K_{bY}	J_{CY}	K_{cY}	$-3K_{CZ}$			K_{CZ}		O
U																					K_{DY}	$-3K_{DY}$			K_{cY}	J_{DY}	$-3K_{DZ}$				K_{DZ}	O
V																							K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	$-W_Z h_Z \div 6E$
W																							K_{AZ}				$-3K_{AZ}$	J_{AZ}	K_{aZ}			O
X																								K_{BZ}			$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}		O
Y																									K_{CZ}		$-3K_{CZ}$		K_{bZ}	J_{CZ}	K_{cZ}	O
Z																										K_{DZ}	$-3K_{DZ}$			K_{cZ}	J_{DZ}	O

TABLE 9.

GENERAL EQUATIONS FOR AN UNSYMMETRICAL FOUR-SPAN BENT
ANY NUMBER OF STORIES HIGH.

Equation	Left-Hand Member of Equation																																					Right-Hand Member of Equation
	Story No. 1						Story No. 2						Story No. 3						Intervening Stories	Story No. X (Second from Top)						Story No. Y (Next to Top)						Story No. Z (Top)						
	R_1	θ_{A1}	θ_{B1}	θ_{C1}	θ_{D1}	θ_{E1}	R_2	θ_{A2}	θ_{B2}	θ_{C2}	θ_{D2}	θ_{E2}	R_3	θ_{A3}	θ_{B3}	θ_{C3}	θ_{D3}	θ_{E3}		R_X	θ_{AX}	θ_{BX}	θ_{CX}	θ_{DX}	θ_{EX}	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	θ_{DY}	θ_{EY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	θ_{DZ}	θ_{EZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																																					
A	$-N_1$	K_{A1}	K_{A1}	K_{C1}	K_{D1}	K_{E1}																																$-W_1 h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}				$-3K_{A2}$	K_{A2}																														O
C	$-3K_{B1}$	K_{a1}	J_{B1}	K_{b1}			$-3K_{B2}$		K_{B2}																													O
D	$-3K_{C1}$		K_{b1}	J_{C1}	K_{c1}		$-3K_{C2}$			K_{C2}																												O
E	$-3K_{D1}$			K_{c1}	J_{D1}	K_{d1}	$-3K_{D2}$				K_{D2}																											O
F	$-3K_{E1}$				K_{d1}	J_{E1}	$-3K_{E2}$					K_{E2}																										O
G		K_{A2}	K_{B2}	K_{C2}	K_{D2}	K_{E2}	$-N_2$	K_{A2}	K_{B2}	K_{C2}	K_{D2}	K_{E2}																										$-W_2 h_2 \div 6E$
H		K_{A2}					$-3K_{A2}$	J_{A2}	K_{a2}				$-3K_{A3}$	K_{A3}																								O
I			K_{B2}				$-3K_{B2}$	K_{a2}	J_{B2}	K_{b2}			$-3K_{B3}$		K_{B3}																							O
J				K_{C2}			$-3K_{C2}$		K_{b2}	J_{C2}	K_{c2}		$-3K_{C3}$			K_{C3}																						O
K					K_{D2}		$-3K_{D2}$			K_{c2}	J_{D2}	K_{d2}	$-3K_{D3}$				K_{D3}																					O
L						K_{E2}	$-3K_{E2}$				K_{d2}	J_{E2}	$-3K_{E3}$					K_{E3}																				O
	Similar equations for intervening stories																																					
O																					K_{AY}	K_{BY}	K_{CY}	K_{DY}	K_{EY}	$-N_Y$	K_{AY}	K_{BY}	K_{CY}	K_{DY}	K_{EY}							$-W_Y h_Y \div 6E$
P																					K_{AY}					$-3K_{AY}$	J_{AY}	K_{aY}				$-3K_{AZ}$	K_{AZ}					O
Q																						K_{BY}				$-3K_{BY}$	K_{aY}	J_{BY}	K_{bY}			$-3K_{BZ}$		K_{BZ}				O
R																							K_{CY}			$-3K_{CY}$		K_{bY}	J_{CY}	K_{cY}		$-3K_{CZ}$			K_{CZ}			O
S																								K_{DY}		$-3K_{DY}$			K_{cY}	J_{DY}	K_{dY}	$-3K_{DZ}$				K_{DZ}		O
T																									K_{EY}	$-3K_{EY}$				K_{dY}	J_{EY}	$-3K_{EZ}$					K_{EZ}	O
U																											K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	K_{EZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	K_{EZ}	$-W_Z h_Z \div 6E$
V																											K_{AZ}					$-3K_{AZ}$	J_{AZ}	K_{aZ}				O
W																												K_{BZ}				$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}			O
X																													K_{CZ}			$-3K_{CZ}$		K_{bZ}	J_{CZ}	K_{cZ}		O
Y																														K_{DZ}		$-3K_{DZ}$			K_{cZ}	J_{DZ}	K_{dZ}	O
Z																															K_{EZ}	$-3K_{EZ}$				K_{dZ}	J_{EZ}	O

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TABLE 10.

GENERAL EQUATIONS FOR AN UNSYMMETRICAL FIVE-SPAN BENT
ANY NUMBER OF STORIES HIGH.

Left Hand Member of Equation — Coefficients of Unknown Slopes and Ratios of Deflection to Story Height

Equation	Story No. 1							Story No. 2						
	R_1	θ_{A1}	θ_{B1}	θ_{C1}	θ_{D1}	θ_{E1}	θ_{F1}	R_2	θ_{A2}	θ_{B2}	θ_{C2}	θ_{D2}	θ_{E2}	θ_{F2}
A	$-N_1$	K_{A1}	K_{B1}	K_{C1}	K_{D1}	K_{E1}	K_{F1}							
B	$-3K_{A1}$	J_{A1}	K_{a1}					$-3K_{A2}$	K_{A2}					
C	$-3K_{B1}$	K_{a1}	J_{B1}	K_{b1}				$-3K_{B2}$		K_{B2}				
D	$-3K_{C1}$		K_{b1}	J_{C1}	K_{c1}			$-3K_{C2}$			K_{C2}			
E	$-3K_{D1}$			K_{c1}	J_{D1}	K_{d1}		$-3K_{D2}$				K_{D2}		
F	$-3K_{E1}$				K_{d1}	J_{E1}	K_{e1}	$-3K_{E2}$					K_{E2}	
G	$-3K_{F1}$					K_{e1}	J_{F1}	$-3K_{F2}$						K_{F2}
H		K_{A2}	K_{B2}	K_{C2}	K_{D2}	K_{E2}	K_{F2}	$-N_2$	K_{A2}	K_{B2}	K_{C2}	K_{D2}	K_{E2}	K_{F2}
I		K_{A2}						$-3K_{A2}$	J_{A2}	K_{a2}				
J			K_{B2}					$-3K_{B2}$	K_{a2}	J_{B2}	K_{b2}			
K				K_{C2}				$-3K_{C2}$		K_{b2}	J_{C2}	K_{c2}		
L					K_{D2}			$-3K_{D2}$			K_{c2}	J_{D2}	K_{d2}	
M						K_{E2}		$-3K_{F2}$				K_{d2}	J_{E2}	K_{e2}
N							K_{F2}	$-3K_{F2}$					K_{e2}	J_{F2}
Similar equations for intervening stories														
M														
N														
O														
P														
Q														
R														
S														
T														
U														
V														
W														
X														
Y														
Z														

Equation	Story No. 3							Intervening Stories	Story No. X (Second from Top)						
	R_3	θ_{A3}	θ_{B3}	θ_{C3}	θ_{D3}	θ_{E3}	θ_{F3}		R_X	θ_{AX}	θ_{BX}	θ_{CX}	θ_{DX}	θ_{EX}	θ_{FX}
A															
B															
C															
D															
E															
F															
G															
H															
I	$-3K_{A3}$	K_{A3}													
J	$-3K_{B3}$		K_{B3}												
K	$-3K_{C3}$			K_{C3}											
L	$-3K_{D3}$				K_{D3}										
M	$-3K_{E3}$					K_{E3}									
N	$-3K_{F3}$						K_{F3}								
Similar equations for intervening stories															
M										K_{AY}	K_{BY}	K_{CY}	K_{DY}	K_{EY}	K_{FY}
N										K_{AY}					
O											K_{BY}				
P												K_{CY}			
Q													K_{DY}		
R														K_{EY}	
S															K_{FY}
T															
U															
V															
W															
X															
Y															
Z															

Equation	Story No. Y (Next to Top)							Story No. Z (Top)							Right-Hand Member of Equation
	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	θ_{DY}	θ_{EY}	θ_{FY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	$\theta_{1 \cdot Z}$	θ_{EZ}	θ_{FZ}	
A															$-W_1 h_1 \div 6E$
B															0
C															0
D															0
E															0
F															0
G															0
H															$-W_2 h_2 \div 6E$
I															0
J															0
K															0
L															0
M															0
N															0
Similar equations for intervening stories															
M	$-N_Y$	K_{AY}	K_{BY}	K_{CY}	K_{DY}	K_{EY}	K_{FY}								$-W_Y h_Y \div 6E$
N	$-2K_{AY}$	J_{AY}	K_{aY}					$-3K_{AZ}$	K_{AZ}						0
O	$-3K_{BY}$	K_{aY}	J_{BY}	K_{bY}				$-3K_{BZ}$		K_{BZ}					0
P	$-3K_{CY}$		K_{bY}	J_{CY}	K_{cY}			$-3K_{CZ}$			K_{CZ}				0
Q	$-3K_{DY}$			K_{cY}	J_{DY}	K_{dY}		$-3K_{DZ}$				K_{DZ}			0
R	$-3K_{EY}$				K_{dY}	J_{EY}	K_{eY}	$-3K_{EZ}$					K_{EZ}		0
S	$-3K_{FY}$					K_{eY}	J_{FY}	$-3K_{FZ}$						K_{FZ}	0
T		K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	K_{EZ}	K_{FZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	K_{EZ}	K_{FZ}	$-W_Z h_Z \div 6E$
U		K_{AZ}						$-3K_{AZ}$	J_{AZ}	K_{aZ}					0
V			K_{BZ}					$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}				0
W				K_{CZ}				$-3K_{CZ}$		K_{bZ}	J_{CZ}	K_{cZ}			0
X					K_{DZ}			$-3K_{DZ}$			K_{cZ}	J_{DZ}	K_{dZ}		0
Y						K_{EZ}		$-3K_{EZ}$				K_{dZ}	J_{EZ}	K_{eZ}	0
Z							K_{FZ}	$-3K_{FZ}$					K_{eZ}	J_{FZ}	0

bent was tested and the measured deflections and changes in the slopes were compared with the same quantities calculated by the above equations. As the entire model was cut from a sheet of celluloid the joints were perfectly rigid. The results of the tests are given in Fig. 15. The fact that the measured and computed deflections and changes in the slopes agree very closely indicates that the above analysis is correct.

A solution of a numerical problem illustrating the use of the equations in Table 3 is given in Section VII.

VII. Numerical Problem.

13. *Determination of the Stresses in a Symmetrical Three-Span Twenty-Story Bent.*—Fig. 5 shows a symmetrical three-span bent twenty stories high. The bent resists a horizontal wind load of 30 lb. per sq. ft. on a vertical strip one foot wide. It is required to find the moment and the shear in the columns and girders and the direct stress in the columns.

The properties of the girder and column sections are shown in Table 11, page 48. The equations of Table 3 are applicable. The numerical values of the constants in these equations are given in Table 12. Table 13 contains the equations of Table 3 with the substitution of the numerical values of the constants given in Table 12. The figures given in the right-hand column are coefficients of .0001 as indicated at the head of the column; that is, the right-hand members of the equations are equal to .0001 times the numbers in the right-hand column. This method of writing the right-hand members of the equations obviates the necessity of repeating a large number of ciphers.

The unknown quantities in the equations of Table 13 are eliminated in Table 14. The quantity R_1 appears in the first three equations of Table 13 only. Dividing each of these equations by the coefficient of R_1 gives the three equations A, B, and C of Table 14, in all of which the coefficient of R_1 is equal to plus unity. Combining these latter equations as indicated, that is, subtracting equation B from A to get equation (A — B) and subtracting equation C from B to get equation (B — C), gives two new equations from which R_1 has been eliminated. Reducing the coefficients of the left-hand terms of equations (A — B) and (B — C) to plus unity gives equations 1 and 2 of Table 14. Equations D and E of Table 13 have also been reduced so that the coefficients of the left-hand term of each are equal to plus unity. They are rewritten in their new form as equations D and E in Table 14. Combining the four equations 1, 2, D, and E, as indicated in Table 14, eliminates

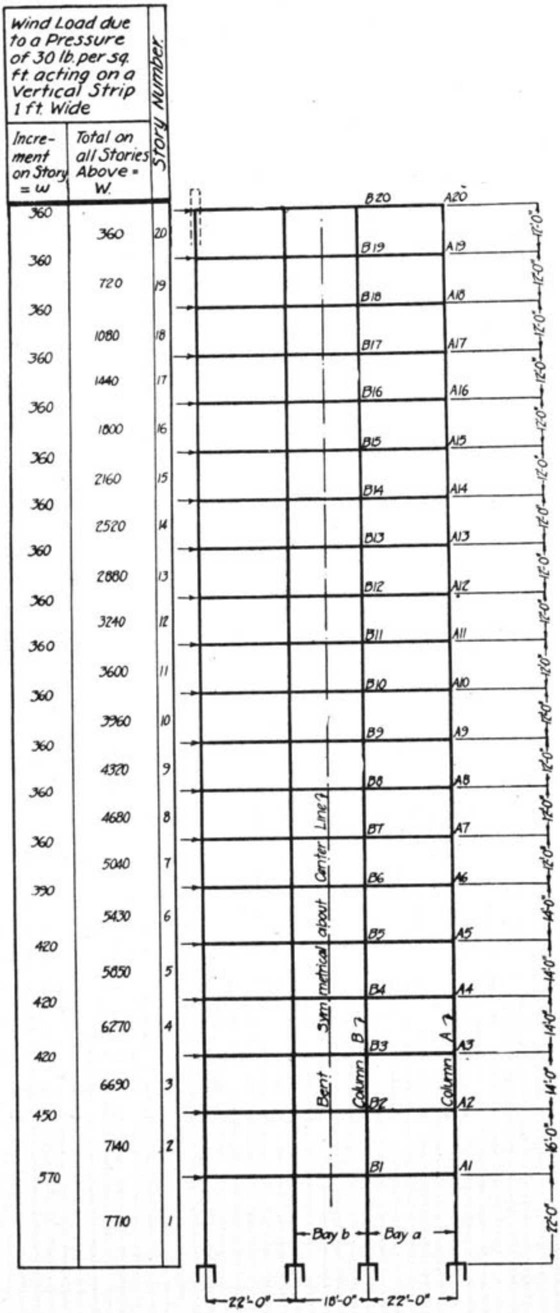

FIG. 5. SYMMETRICAL THREE-SPAN BENT, TWENTY STORIES HIGH.

the left-hand term, θ_{A_1}. By continuing this process all the unknown quantities are eliminated except the last, or $\theta_{B_{20}}$. Its value thus becomes known and is found to be .0338 × .0001 radians. With the value of $\theta_{B_{20}}$ known, the value of $\theta_{A_{20}}$ can be determined from equation 153. With $\theta_{B_{20}}$ and $\theta_{A_{20}}$ both known, the value of R_{20} can be determined from equation 150. The other values of θ and R can be determined in a similar manner.

The process of determining the values of θ and R is given in Table 15. The equations used are taken from Table 14.

As in Tables 13 and 14, the right-hand member is equal to the number in the column at the right multiplied by .0001. To illustrate, equation 150 is

$$R_{20} - .2702\,\theta_{A_{20}} - .2993\,\theta_{B_{20}} = .0735 \times .0001.$$

The left-hand term of the left-hand member of each equation is the unknown which is to be determined. The first line in each group is the algebraic form of the equation; and the successive lines are the numerical values of the corresponding terms. For example, equation 155 is

$$\theta_{B_{20}} = .0338 \times .0001 \text{ or } .00000338.$$

Again, equation 153 is:

$$\theta_{A_{20}} - .0443\,\theta_{B_{20}} = .0502 \times .0001.$$

From the preceding equation

$$.0443\,\theta_{B_{20}} = .0443 \times .00000338 = .0015 \times .0001.$$

The sign of this quantity as a term of the left-hand member was negative, and the term will, therefore, be positive after it has been transferred.

Equation 153 now becomes

$$\theta_{A_{20}} = .0502 \times .0001 + .0015 \times .0001 = .0517 \times .0001,$$

as indicated in the last line of the group containing equation 153. Equation 150 contains R_{20}, $\theta_{A_{20}}$, and $\theta_{B_{20}}$; but $\theta_{A_{20}}$ and $\theta_{B_{20}}$ have been determined by equations 153 and 155, respectively, therefore R_{20} can be determined by equation 150. Similarly, equation 147, containing $\theta_{B_{19}}$ and the three known quantities, R_{20}, $\theta_{A_{20}}$, and $\theta_{B_{20}}$ can be used to determine $\theta_{B_{19}}$. Equation 147 differs from the preceding equations in that it contains both positive and negative terms in the left-hand member. The second term of the left-hand member, being negative, when transferred, is added to the right-hand member. The sum of the two quantities is .0890 × .0001. Since the third and fourth terms of the left-hand member are positive, their sum, when transferred, is subtracted from the quantity above. The final form of the equation is: $\theta_{B19} = .0835 \times .0001$. In a similar manner the other slopes, θ, and the ratios of the deflection to the story height, R, can be determined. θ is expressed in radians, and R is an abstract quantity.

The tabulation of the calculations as shown in Tables 14 and 15 facilitates the solution of a large number of equations. There are however a number of practical considerations which should be borne in mind when solving a problem by this method.

An error at any point affects all of the calculations which follow. It is therefore desirable to have two computers carry on the work simultaneously and compare results at frequent intervals in order to avoid the loss of time due to errors.

A number of combinations can be made from each group of equations. It is desirable to combine the equations so as to make the coefficient of the left-hand term of the left-hand member of the resulting equation as large as possible compared with the coefficients of the other terms in the equation. To illustrate this point, consider equations 25, 26, and M of Table 14. If equation M had been subtracted from equation 26, the coefficient of θ_{B4} in the resulting equation would have been .1463 and the coefficient of R_5 would have been 2.9027. The latter is about twenty times as great as the former. Any small error in the actual value of the former coefficient would be a large percentage of error. This same percentage of error would be introduced into the latter coefficient and produce a large actual error. With the combination of equations 25 and M as used, the coefficient of θ_{B4} is 1.0695 and of R_5, 3.4313. The latter coefficient is only about three times as large as the former and hence the effect of an inaccuracy in the value of the former upon the latter is correspondingly smaller than in the first case.

It is possible in some cases to combine the equations in such a way as to give equations which are algebraically independent, but which are numerically nearly identical. Combining such equations gives results which are likely to be inaccurate. Any tendency of the equations to become identities can be avoided by changing the order in which they are combined.

In getting the values of the remaining unknown quantities after one has been determined, each unknown is expressed in terms of the known quantities. By referring to Table 14 it is seen that in the first equation of a group of equations, any one of which could be used to get the value of an unknown, the coefficient of the quantity which is to be determined is larger than the coefficients of the known quantities. If this equation is used, any inaccuracies in the known quantities will affect the accuracy of the results less than they would if an equation had been used in which the coefficient of the unknown is less than the coefficient of the known quantities. The various equations available should be examined and the one used which causes the least accumulation of inaccuracies.

In order to determine the extent to which inaccuracies in the calculations accumulate, two independent sets of calculations were made; one set was made on a 20-inch slide rule and the other on a Fuller's cylindrical slide rule. The two sets of calculations were compared at frequent intervals, and mistakes were corrected; but no adjustment of inaccuracies was made. The maximum variation in the slopes and deflections as determined in the two sets of calculations was very small. This indicates that the calculations can be made with a slide rule without greater inaccuracies than are permissible.

The moments at the ends of the columns and girders can be determined by substituting the values of the deflections and the slopes, given in Table 15, in equations A and B of Section V. To facilitate this work, the quantities which occur in these equations are given in Tables 16 and 17. The values of R and θ taken from Table 15 are given in the second, third, and fourth columns of Table 16; and the functions of the values of R and θ which occur in the equations are given in the remaining columns of the same table. The values of K taken from Table 12 are given in the second, third, fourth, and fifth columns of Table 17; and the functions of the values of K which occur in the equations are given in the remaining columns of the table. The moments at the ends of the columns and girders given by the fundamental equation, $M_{AB} = 2EK\,(2\theta_A + \theta_B - 3R)$, are the products of two factors, one of which is given in Table 16 and the other in Table 17. The values of the moments are given in Table 18.

The shear in any member is equal to the algebraic sum of the moments at the two ends of the member divided by its length. The direct stress at any section in column A is equal to the algebraic sum of the shears in the girders in bay a above the section. The direct stress at any section in column B is equal to the algebraic sum of the shears in the girders in bay a plus the algebraic sum of the shears in the girders in bay b above the section. The shears in the columns and girders and the direct stresses in the columns are given in Table 19.

The sum of all of the moments at the top and the bottom of all the columns in a story is equal to the total shear on the story multiplied by the story height. The algebraic sum of the moments in the girders on the two sides of a column is equal to the algebraic sum of the moments in the columns on the two sides of the girder.

The total shear on a story multiplied by the story height and the algebraic sum of all of the moments at the top and the bottom of all columns of a story, as given in Table 18, are given in the second and third columns of Table 20. The algebraic sum of the moments in column A at sections immediately above and below the girders and the moments in the girders in bay a at sections adjacent to column A, are given in the fourth and fifth columns of Table 20. The algebraic sum of the moments in column B at sections immediately above and below the girders and the algebraic sum of the moments in the girders in bays a and b at sections adjacent to column B, are given in the sixth and seventh columns of Table 20. The accuracy of the computations can be checked by comparing the two values for the same quantity as given in adjacent columns of Table 20. To illustrate, for the sixth story, the total shear in the story multiplied by the story height is 913,000 inch-pounds, and the sum of the moments at the tops and bottoms of all columns in the story is 914,400 inch-pounds. For a perfect check, the two quantities would be equal. An inspection of the table shows that the values check very closely.

The moments given in Table 18 are due to a horizontal wind load of 30 lbs. per sq. ft. acting on a vertical strip one foot wide. To obtain the moments due to the wind load on any portion of the building, multiply the moments in Table 18 by the width in feet of the portion of the building on which the wind load acts.

VIII. Approximate Methods.

14. *Nomenclature of Methods.*—The method used in Section VII to determine the stresses in a symmetrical three-span twenty-story bent can be used in the actual design of a building, but a shorter method is desirable. The writers propose an approximate method which is much shorter than the method used in Section VII and which they believe is sufficiently accurate to be used in the actual design of buildings. For the sake of convenience in reference, this approximate method will be designated as the "Proposed Approximate Method" as distinguished from the method used in Section VII, which will be designated as the "Slope-Deflection Method." The slope-deflection method can be modified to advantage under certain special conditions. Such modifications

will be designated as "Modifications of the Slope-Deflection Method."

15. *Proposed Approximate Method.*—The proposed approximate method is based upon the following assumptions in addition to those used in the slope-deflection method.

1. The changes in the slope at the top of a column in the story above and in the story below the one in which the stresses are to be determined, are equal to the change in the slope at the top of the corresponding column in the latter story.

2. The ratio of the deflection to the length of the columns in the story above the one in which the stresses are to be determined, is equal to the ratio of the deflection to the length of the columns in the latter story.

In other words, in determining the stresses in the second story, θ_{A_1} and θ_{A_3} are assumed to be equal to θ_{A_2}; θ_{B_1} and θ_{B_3} are assumed to be equal to θ_{B_2}; and R_3 is assumed to be equal to R_2. Also, in figuring the stresses in the third story θ_{A_2} and θ_{A_4} are assumed to be equal to θ_{A_3}; θ_{B_2} and θ_{B_4} are assumed to be equal to θ_{B_3}; and R_4 is assumed to be equal to R_3. This does not mean, however, that the values of θ_{A_2}, θ_{B_2}, and R_2 used in determining the stresses in the second story are equal to the values of θ_{A_3}, θ_{B_3}, and R_3 respectively, used in determining the stresses in the third story.

An examination of the equations in Table 3 shows that, if assumptions 1 and 2 were true, three equations containing only three unknown quantities could be written for each story of the bent. To illustrate, consider the equations for the second story. In accordance with assumptions 1 and 2,

$$\theta_{A_1} = \theta_{A_2} = \theta_{A_3}$$
$$\theta_{B_1} = \theta_{B_2} = \theta_{B_3}$$
$$R_2 = R_3$$

Substituting θ_{A_2} for θ_{A_1} and θ_{A_3}, θ_{B_2} for θ_{B_1} and θ_{B_3}, and R_2 for R_3 in equations D, E, and F of Table 3 gives:

$$-N_2R_2 + 4K_{A_2}\theta_{A_2} + 4K_{B_2}\theta_{B_2} = -\frac{W_2h_2}{6E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$-3\,(K_{A_2} + K_{A_3})\,R_2 + (K_{A_2} + J_{A_2} + K_{A_3})\,\theta_{A_2} + K_{a_2}\theta_{B_2} = 0 \ldots\ldots\ldots (2)$$

$$-3\,(K_{B_2} + K_{B_3})\,R_2 + K_{a_2}\theta_{A_2} + (K_{B_2} + J_{B_2} + K_{b_2} + K_{B_3})\,\theta_{B_2} = 0 \ldots (3)$$

These equations have been written for the second story. Similar equations can be written for any other story by making the proper changes in the subscripts.

Equations 1, 2, and 3, obtained from the equations in Table 3, can be used in determining the stresses in a symmetrical three-span bent any number of stories high. If similar changes are made in the equations of Tables 1 to 10 inclusive, groups of equations can be obtained, one for each bent, which can be used to determine the stresses in any story of *symmetrical* bents of from one to five spans and also of *unsymmetrical* bents of from one to five spans.

The sum of the moments at the tops and bottoms of all columns of a story is equal to the total shear in the story multiplied by the story height. The distribution of this moment to the ends of the columns depends upon: first, the ratio of the K of column A to the K of column B; second, the ratio of the K of column A to the K of girder a; and third, the ratio of the K of girder a to the K of girder b.

The distribution of the moment was determined in a number of bents for which the ratios of the K of column A to the K of column B, the K of column A to the K of girder a, and the K of girder a to the K of girder b had different values. Diagrams showing the distribution of the moment in these bents also give the distribution of the moment in other bents. The curves in Figs. 6, 7, and 8 show the distribution of the moment in symmetrical three-span bents. In Fig. 6, the curves in group I show the moment at the top and the bottom of column A in bents in which the K of column A and the K of column B are equal. The abscissae represent the ratios of the K of girder a to the K of girder b; and the ordinates represent the moment at the top and the bottom of column A in per cent of $W \times h$. Beginning at the top and reading down, the curves are for bents for which the ratio of the K of column A to the K of girder a equals 0.5, 1, 2, and 4 respectively. The moment in column A of a bent for which the ratio of the K of column A to the K of girder a has any intermediate value can be determined by interpolation.

The curves in groups II, III, IV, and V, of Fig. 6, when used with the curves in group I, give the moment at the top and the bottom of column A in bents in which the K of column A and the K of column B are not equal. The curves in groups II, III, IV, and V are for bents for which the ratio of the K for column A to the K for girder a equals 4, 2, 1, and 0.5, respectively. Beginning at the left of each group and reading to the right, the curves are for bents for which the ratio of the K for column A to the K for column B equals 0.5, 1, and 2, respectively.

The moment at the top and the bottom of column A can be obtained from the diagram in Fig. 6 in the following manner:

First consider the moment in a bent in which the K of column A is

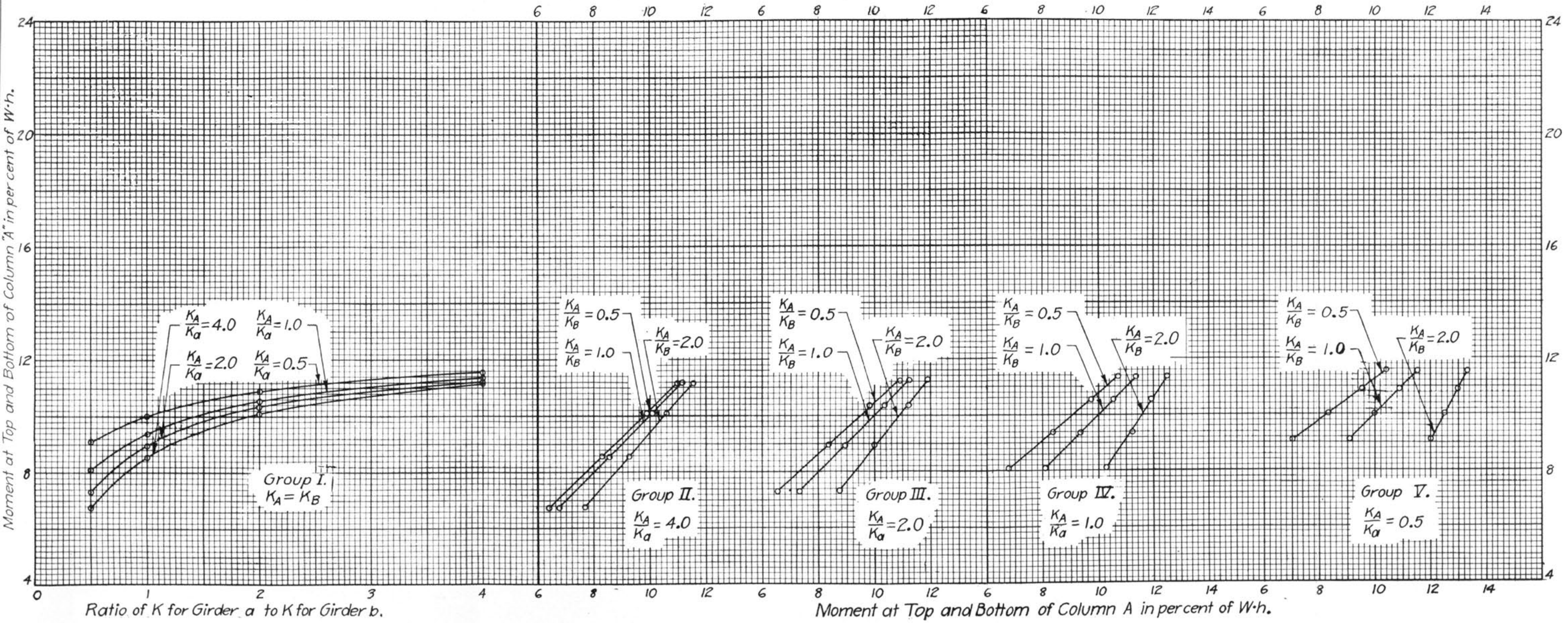

Fig. 6. Approximate Moment at the Top and the Bottom of Column *A*.

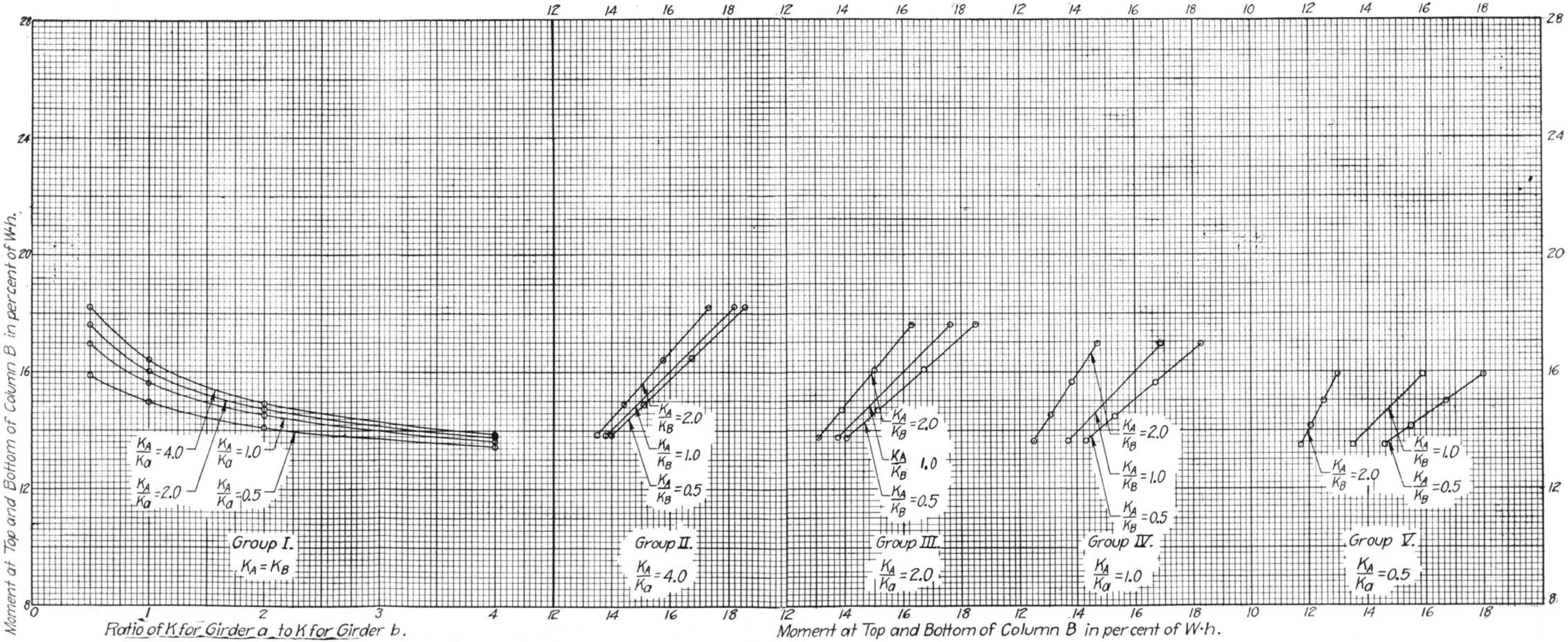

Fig. 7. Approximate Moment at the Top and the Bottom of Column *B*.

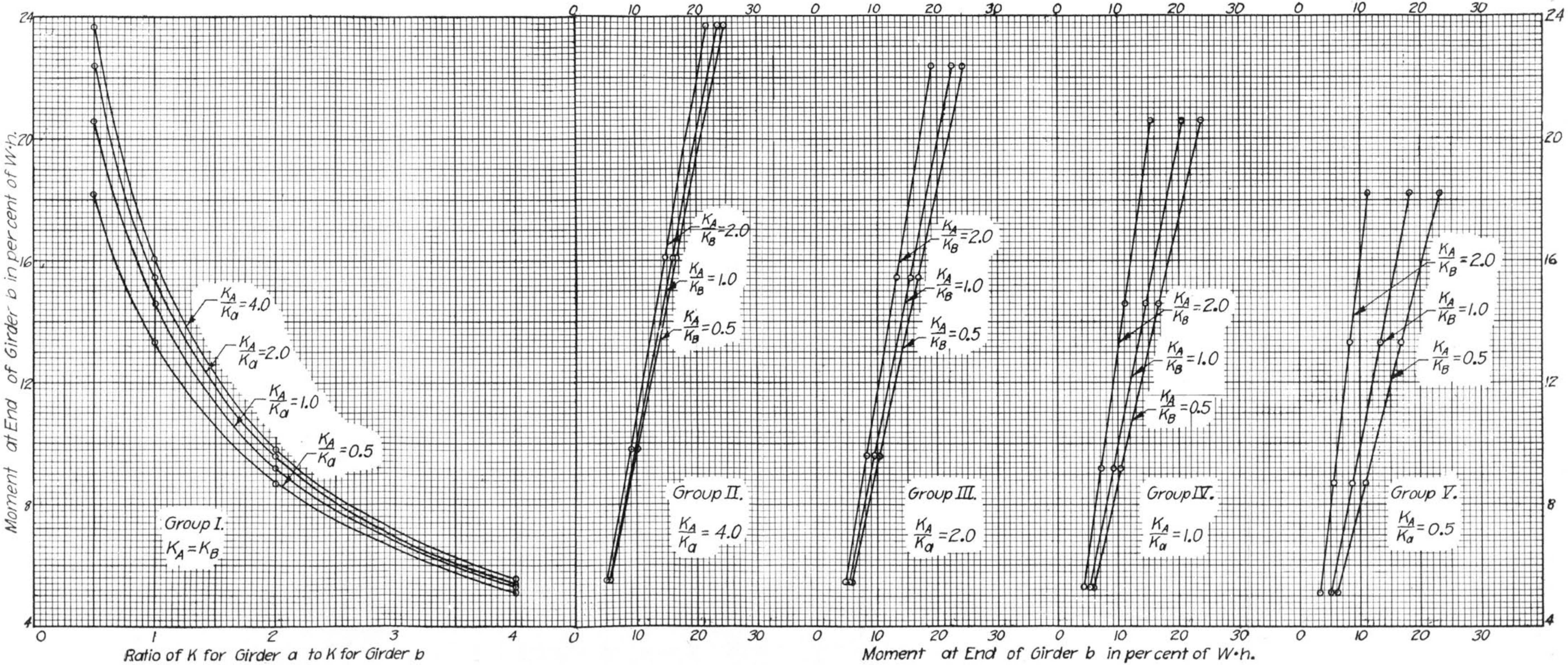

Fig. 8. Approximate Moment at the Ends of Girder b.

equal to the K of column B. Determine the value of $\frac{K_a}{K_b}$ and of $\frac{K_A}{K_a}$. Use the curves in group I. Trace a vertical line whose abscissa equals $\frac{K_a}{K_b}$ until it intersects the curve corresponding to the value of $\frac{K_A}{K_a}$. Project the point of intersection horizontally to the left and read the moment in per cent of $W \times h$ from the vertical scale. As W and h are known quantities the moment is determined.

Next consider the moment in a bent in which the K of column A and the K of column B are not equal. Determine the value of $\frac{K_a}{K_b}$, $\frac{K_A}{K_a}$, and $\frac{K_A}{K_B}$. First use the curves of group I. Trace a vertical line whose abscissa equals $\frac{K_a}{K_b}$ until it intersects the curve corresponding to the value of $\frac{K_A}{K_a}$. Project the point of intersection horizontally to the right to the group of curves corresponding to the value of $\frac{K_A}{K_a}$ until it intersects the particular curve of this group corresponding to the value of $\frac{K_A}{K_B}$. Project this intersection point vertically downward and read the moment in per cent of $W \times h$ from the horizontal scale.

Similarly, the moments at the top and the bottom of column B can be obtained from Fig. 7, and the moment at the end of girder b can be obtained from Fig. 8. It should be noted, however, that the moment in girder b depends equally upon the $W \times h$ in the story above and in the story below the girder. This being true, in getting the moment in girder b, the average of the $W \times h$ for the two stories should be used.

The moment at the right end of girder a balances the sum of the moments in column A just above and below the girder and is equal to their algebraic sum. The moment at the left end of girder a balances the sum of the moments at the right end of girder b together with the moments in column B just above and below girder a and is equal to their algebraic sum. It is therefore possible to obtain the moment at the ends of all members in a bent from the curves in Figs. 6, 7, and 8, subject, of course, to the error due to the use of assumptions 1 and 2 of this Section.

The curves in Figs. 6, 7, and 8 show that a large change in the ratio of the K of one member to the K of another member causes a relatively small change in the distribution of the moments in the bent.

16. *Numerical Problem.*—To illustrate the use of these curves the solution of a problem is presented.

The seventh story of a symmetrical three-span bent is 20 ft. high, and is subjected to a shear of 3,000 lbs. The eighth story of the same bent is 20 ft. high, and is subjected to a shear of 2,500 lbs. The properties of the members in the seventh and eighth stories are as follows:

$K_A = 30$ in.^{3}; $K_B = 40$ in.^{3}; $K_a = 20$ in.^{3}; and $K_b = 16$ in.^{3}. It is required to find the moments at the ends of all members in the seventh story.

$W \times h = 3{,}000 \times 20 \times 12 = 720{,}000$ in. lb. in the seventh story.
$W \times h = 2{,}500 \times 20 \times 12 = 600{,}000$ in. lb. in the eighth story.

$$\frac{K_A}{K_B} = \frac{30}{40} = .75$$

$$\frac{K_A}{K_a} = \frac{30}{20} = 1.5$$

$$\frac{K_a}{K_b} = \frac{20}{16} = 1.25$$

To get the moment in column A use Fig. 6. At the left of the figure trace the ordinate whose abscissa is 1.25 to a point half way between the two middle curves ($\frac{K_a}{K_A} = 1.5$, which is half way between 1 and 2), and project this point horizontally to a point half way ($\frac{K_A}{K_B} =$ .75, which is half way between .5 and 1.0) between the two left-hand curves of Group II, and also to a point half way between the two left-hand curves of Group III. The abscissa of the former point is 9.35 per cent, and of the latter point 9.15 per cent. As $\frac{K_A}{K_a}$ is equal to 1.5 or the average of 1 and 2, the moment at the top and the bottom of column A in the seventh story, M_{A7}, is the average of 9.35 per cent and 9.15 per cent or 9.25 per cent of $W \times h$, that is, $M_{A7} = .0925 \times 720{,}000 = 66{,}700$ in. lb. The moment, M_{A8}, at the top and the bottom of column A in the eighth story is $.0925 \times 600{,}000$, or $M_{A8} = 55{,}500$ in. lb. Similarly, the moment at the top and bottom of column B, M_{B7}, is 15.75 per cent of $W \times h$, that is, $M_{B7} = .1575 \times 720{,}000 = 113{,}500$ in. lb. in the seventh story; and $M_{B8} = .1575 \times 600{,}000 = 94{,}400$ in. lb. in the eighth story. The moment M_{b7}, at the end of girder b at the top of the seventh story is 13.85 per cent of $W \times h$, that is,

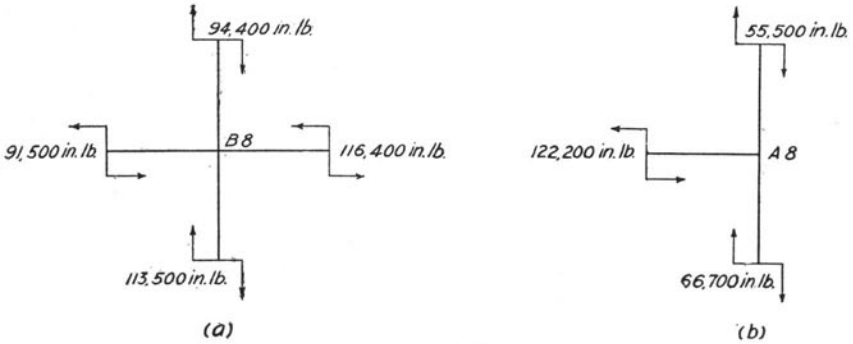

FIG. 9. MOMENTS ACTING AT POINTS *A8* AND *B8* OF A SYMMETRICAL THREE-SPAN BENT.

$$M_{b_7} = .1385 \left[\frac{720{,}000 + 600{,}000}{2} \right] = 91{,}500 \text{ in. lb.}$$

The moment at the right end of girder *a* at the top of the seventh story is equal to the sum of 66,700 in. lb., the moment at the top of column *A* in the seventh story, and 55,500 in. lb., the moment at the bottom of column *A* in the eighth story, or 122,200 in. lb. (see Fig. 9b). The moment at the left end of girder *a* is equal to the sum of 113,500 in. lb., the moment at the top of column *B* in the seventh story, and 94,400 in. lb., the moment at the bottom of column *B* in the eighth story, less 91,500 in. lb., the moment in the end of girder *b;* that is, the moment at the left end of girder *a* is 116,400 in. lb. (see Fig. 9a).

A comparison of the moments in a bent as obtained by the proposed approximate method and by the slope-deflection method is given in Tables 23 to 26 inclusive.

17. *Modifications of the Slope-Deflection Method.*—The writers made a study to determine the effect of a change in the section of one member of a bent upon the moment in the other members. The moments were determined in a number of bents in which $K = 1$ for all members except the one whose effect upon the distribution of the moment was to be studied. The quantity $\frac{Wh}{6E}$ was taken equal to 1 for all stories and for all bents. The K of column B was given successively the values 0.5, 1, 2, and 4; and the corresponding moment at the ends of columns A and B and at the ends of girders *a* and *b* was calculated

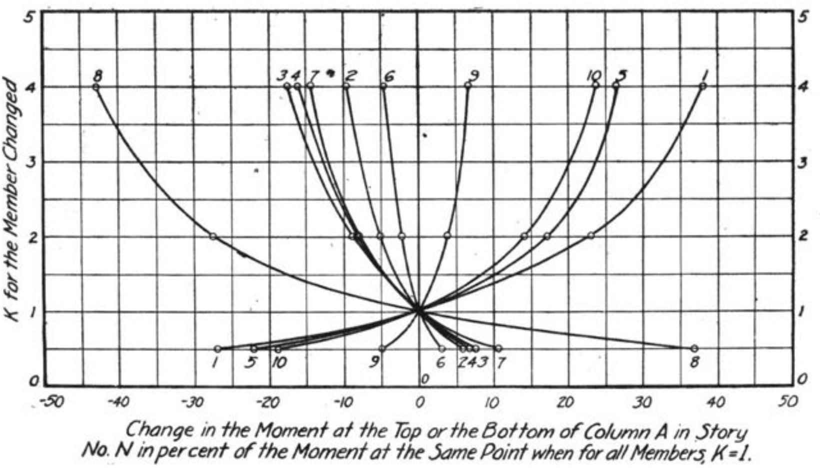

FIG. 10. DIAGRAM SHOWING CHANGE IN THE MOMENT AT THE TOP AND THE BOTTOM OF COLUMN A OF A SYMMETRICAL THREE-SPAN BENT DUE TO A CHANGE OF K IN THE OTHER MEMBERS.

KEY TO THE DIAGRAM IN FIG. 10.

Member Changed	Designation of the Curve Showing the Change in the Moment at	
	Top of Column A	Bottom of Column A
Girder a in Story No. N	1	2
" " No. (N—1)	2	1
Girder b " " No. N	3	4
" " No. (N—1)	4	3
Column A " " No. N	5	5
" " No. (N—1)	6	7
" " No. (N+1)	7	6
Column B " " No. N	8	8
" " No. (N—1)	9	10
" " No. (N+1)	10	9

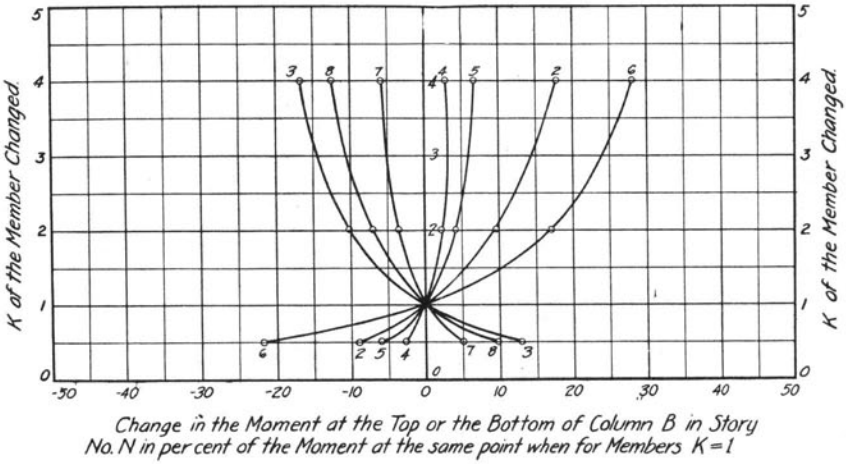

FIG. 11. DIAGRAM SHOWING CHANGE IN THE MOMENT AT THE TOP AND THE BOTTOM OF COLUMN *B* OF A SYMMETRICAL THREE-SPAN BENT DUE TO A CHANGE OF *K* IN THE OTHER MEMBERS.

KEY TO THE DIAGRAM IN FIG. 11.

Member Changed	Designation of the Curve Showing the Change in the Moment at: Top of Column *B*	Designation of the Curve Showing the Change in the Moment at: Bottom of Column *B*
Girder *a* in Story No. N	—	1
" " No. (N—1)	1	—
Girder *b* " " No. N	2	—
" " No. (N—1)	—	2
Coulmn *A* " " No. N	3	3
" " No. (N—1)	4	5
" " No. (N+1)	5	4
Column *B* " " No. N	6	6
" " No. (N—1)	7	8
" " No. (N+1)	8	7

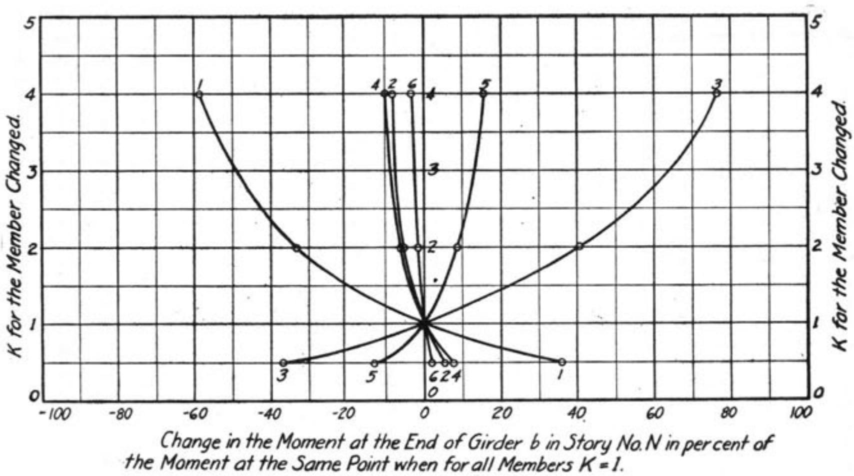

FIG. 12. DIAGRAM SHOWING CHANGE IN THE MOMENT AT THE END OF GIRDER b OF A SYMMETRICAL THREE-SPAN BENT DUE TO CHANGE OF K IN THE OTHER MEMBERS.

KEY TO THE DIAGRAM IN FIG. 12.

Member Changed	Designation of the Curve Showing the Change in the Moment at the End of Girder b
Girder a in Story No. N	1
" " No. (N—1)	2
" " No. (N+1)	2
Girder b " " No. N	3
Column A " " No. N	4
" " No. (N+1)	4
Column B " " No. N	
" " No. (N—1)	6
" " No. (N+1)	5

in the story in which the K of column B was changed, and also in the stories immediately above and below that story. Similarly, the K of column A, the K of girder a and the K of girder b were given successively the values 0.5, 1, 2, and 4, and the corresponding values of the moment at the ends of the girders and the columns were determined in the particular story and also in the stories immediately above and below that story. The effect of the changes in the values of the K of the members upon the moment is given in Figs. 10, 11, and 12.

Fig. 10 shows the changes in the moment at the top and the bottom of column A. The abscissae are changes in the moment expressed in per cent of the moment in a bent for which the K of all members is equal to 1. Increases in the moment are laid off to the right of the origin, and decreases are laid off to the left. The ordinates represent the values of the K in the member changed. The story in which the moment was calculated is designated as story No. N, the story above is designated as story No. $(N+1)$, and the story below No. $(N-1)$. Each curve shows the change in the moment at the top or the bottom of column A due to a change in the value of the K of some member. The number of the curve which shows the change in the moment in column A due to a change in the K of any particular member can be obtained from the key to the diagram in Fig. 10. For example, curve No. 3 shows the change in the moment at the top of column A due to a change in the K of girder b in story No. N. If the K of girder b is made equal to 2, the moment at the top of column A is 8.9 per cent less than when the K of girder b is equal to 1. Curve No. 3 also shows the change in the moment at the bottom of column A due to a change in the K of girder b in story No. $(N-1)$. If the K for girder b in story No. $(N-1)$ is equal to 2, the moment at the bottom of column A is 8.9 per cent less than when the K for girder b is equal to 1.

Similarly, the changes in the moment in column B due to changes in the K of the members of the bent are given in Fig. 11 and the changes in the moment in girder b are given in Fig. 12.

Fig. 10 shows that the moment in column A of any story not only depends upon the value of the K of the members in the same story but depends also upon the value of the K of the members in the adjacent stories. Figs. 11 and 12 show that the same statement is true relative to the moment in column B and girder b. This being true, any approximate method which considers only the members in the story in which the stresses are to be determined can not give accurate results except in bents which have no very sudden changes in the sections of the columns and girders.

Where there are sudden changes in the column and girder sections, the proposed approximate method is not accurate. In such cases it is sometimes possible to use a modification of the slope-deflection method and obtain quite accurate results with a comparatively small expenditure of labor. This modification of the slope-deflection method may be made in either of two ways, as follows: (a), the bent may be divided by a horizontal plane between any two adjacent floors, and the lower part be treated independently of the upper part; or (b), the bent may be divided by two horizontal planes any number of stories apart, and the portion between these planes may be treated independently of the other portions of the building. These modifications will be considered in detail.

(a) The wind stresses in the top stories of a bent are comparatively small and their exact determination is not important. An examination of the equations of Table 15 shows that the coefficient of the quantity to be determined is greater than the coefficients of the quantities in which the unknown is expressed. This being true, errors in the slopes and the deflection of one story have but little effect upon the slopes and deflections of the succeeding stories. Further examination of the same table shows that there is a comparatively small difference between the change in the slope at the top of a column in one story and the change in the slope at the top of the same column in the story below. If the two changes in slope are assumed to be equal, the error introduced is small. Bearing these facts in mind, consider again the twenty-story bent of Section VII. If θ_{A12} and θ_{B12} are assumed equal to θ_{A11} and θ_{B11} respectively in equations f, g, and h of Table 13, the first 34 equations will contain 34 unknown quantities which can be determined by the method used in Tables 14 and 15. That is, assuming that the changes in the slopes of the columns in one story are equal to the changes in the slopes of the same columns in the story below, is equivalent to dividing the bent into two parts; and the equations for the lower part can be solved independently of those of the upper part. If the slopes which are assumed equal are not really equal, the calculated slopes and deflections in the top story of the lower part will not be exact, but the results of a number of calculations show that the error in the slopes and the deflection in the next to the top story is so small that it may be neglected. This being true, if, in the actual design of the twenty-story bent of Section VII, it is considered unnecessary to calculate the wind stresses in the top ten stories, the bottom eleven stories may be treated as a complete bent and the stresses determined by the method outlined above, thereby decreasing the work. The stresses in the bottom ten stories will be quite exact. The bent could have been divided at any other story without affecting the accuracy of the results.

If there are sudden changes in the members the bent can be divided above the story in which the change occurs, and the stresses in the lower part can be determined by the method outlined above.

Table 23 shows that the largest errors in the proposed approximate method are in the first story. The moments can be determined in the first story by the modification of the slope-deflection method as follows:

Assume that the slopes in the third story are equal to the corresponding slopes in the second story. The first seven equations of Table 13 will then contain only seven unknown quantities. The solution of these equations is given in Tables 21 and 22. The changes in the slopes and the deflection for the first story as given in Table 22 agree very closely with the values for the same quantities given in Table 15, and the moments, being functions of the slopes and deflection, will also agree very closely with the moments given in Table 18.

(b) The moments in any particular story may be determined if there is some story below the one in question in which it can be seen by inspection that the changes in the slopes at the tops and the bottoms of the columns are equal. Suppose the moments are to be determined in the tenth story of the bent shown in Fig. 5, and that it is apparent from inspection that $\theta_{A8} = \theta_{A9}$, and $\theta_{B8} = \theta_{B9}$. Then assume that $\theta_{A12} = \theta_{A11}$, and $\theta_{B12} = \theta_{B11}$. The ten equations, Y to h inclusive of Table 13, will contain ten unknown quantities. If the slopes in the eighth and ninth stories, which have been assumed to be equal, are equal then the slopes and the deflection, and therefore also the moments, in the tenth story will be quite accurate, but if the slopes *below* the story in question which are assumed to be equal, are not really equal, the required moments will not be exact. That is, a difference between the slopes *below* the story in question affects the results, whereas a difference between the slopes *above* the story does not materially affect the results.

18. *Application of the Proposed Approximate Method and Modification of the Slope-Deflection Method.*—To obtain the moment in a bent in which there are sudden changes in the sections, the following com-

bination of methods can sometimes be used to advantage. Use the proposed approximate method for obtaining the moment in the portion of the bent in which there are no sudden changes in the sections of the members. Use a modification of the slope-deflection method to get the moments in the bottom story and in any intermediate stories in which sudden changes in the members occur. The results thus obtained will be sufficiently accurate to be used in the actual designs of buildings, and the amount of work required will not be excessive.

IX. Comparison of the Approximate Methods With the Slope-Deflection Method.

19. *Symmetrical Three-Span Bent with Short Middle Span.*—For comparison, the moments in the symmetrical three-span twenty-story bent shown in Fig. 5 were calculated by the five following methods:

1. The slope-deflection method.
2. The proposed approximate method.
3. The three methods described by Mr. Fleming and known as methods I, II, and III.

The moments as determined by these methods are given in Tables 23 and 24. For each story the moments in the upper line are in 1,000 in. lb., and those in the lower line are in per cent of the moments as determined by the slope-deflection method. Tables 23 and 24 show in the case of this bent: first, that the moments determined by methods II and III are very seriously in error; second, that the moment determined by method I and by the proposed approximate method agree very closely with the moment determined by the slope-deflection method except at points where there are sudden changes in the members of the bent; third, that the errors in the moment determined by the proposed approximate method are less for the girders than for the columns.

20. *Symmetrical Three-Span Bent with Long Middle Span.*—The distribution of the moment determined by the slope-deflection method is affected by the ratio of the K of girder a to the K of girder b. As this ratio does not affect the distribution of the moment determined by methods I, II, and III, the accuracy of the latter methods will depend upon the relative values of K for the two girders.

In the bent shown in Fig. 5, the K is less for girder a than for girder b, since the girders have substantially the same sections and girder a is longer than girder b. In order to determine the effect of the relative values of the K of girders a and b upon the accuracy of the approximate methods, the moment in the bottom four stories of the bent shown in Fig. 13, a bent like the one shown in Fig. 5 except that the long and short spans have been interchanged, was determined by each of the five methods mentioned in article 19. The results are given in Tables 25 and 26. The moments determined by the proposed approximate method are as accurate as those given in Tables 23 and 24; whereas the moments determined by method I are inaccurate, and those determined by methods II and III are very inaccurate.

21. *Effect of the Proportions of a Bent upon the Accuracy of Method I.*—In order to determine more fully the effect of the proportions of a bent upon the accuracy of method I, the moments were determined in a number of bents having different proportions, by the slope-deflection method and by method I. These moments are given in Table 27.

In bents 1 to 9 of this table all girders and columns have the same section. The ratio of the moment of inertia of the columns to the moment of inertia of the girders and also the ratio of the moment of inertia of the girder in bay *a* to the moment of inertia of the girder in bay *b*, affects the moment determined by the slope-deflection method but does not affect the moment as determined by method I. If the sections of the columns and girders are not the same, the difference between the moments determined by the two methods might vary even more than Table 27 indicates. Any errors in the moment due to sudden changes in the sections of the bent are in addition to the errors indicated in Table 27.

The difference between the two methods is due to the fact that in method I the direct unit stress in a column is proportional to the distance of the column from the neutral axis of the bent; whereas in the slope-deflection method the stress in a column depends upon the shears in the girders, and the shears in the girders depend upon the changes in the slopes at the ends of the girders and upon the moment of inertia of the girder sections.

22. *Accuracy of the Approximate Methods when the Moment of Inertia of the Girders is Proportional to the Bending Moment.*—In the comparison of the approximate methods with the slope-deflection method given in Tables 23 to 27, the sections of the girders were chosen without reference to the moment to which they were subjected, except the girders in the bent shown in Fig. 5. The girders in this bent were designed to resist the bending moment determined by method I. Another investigation was made to determine the accuracy of the approxi-

Wind load due to a Pressure of 30 lb. per sq. ft. acting on a Vertical Strip 1 ft. Wide. Increment on Story = w	Total on all Stories above = W	Story Number
360	360	20
360	720	19
360	1080	18
360	1440	17
360	1800	16
360	2160	15
360	2520	14
360	2880	13
360	3240	12
360	3600	11
360	3960	10
360	4320	9
360	4680	8
360	5040	7
390	5430	6
420	5850	5
420	6270	4
420	6690	3
450	7140	2
570	7710	1

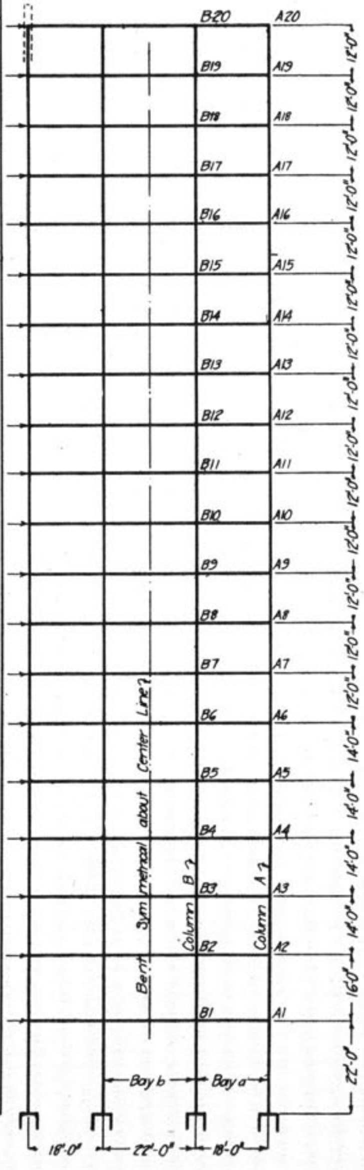

FIG. 13. SYMMETRICAL THREE-SPAN BENT TWENTY STORIES HIGH WITH LONG SPAN AT THE CENTER.

mate methods when applied to bents in which the girders were designed to resist the bending moment due to the wind load as determined by the method which is to be compared with the slope-deflection method. The results of this investigation are given in Tables 28 to 31. In Table 28 the moments of inertia of the sections of the girders are proportional to the bending moments determined by method I. The sections of the columns are equal, and the moment of inertia of the sections of the columns is equal to the moment of inertia of the section of girder *a*. Table 28 shows that method I gives the moment in the columns and girders quite accurately when the spans are equal and the story height does not exceed the length of the span. This statement is true, in general, only when the two column sections are equal and when the moments of inertia of the sections of column *A* and girder *a* are equal.

In considering the merits of the approximate methods, it should be noted that the moment which should be determined with the greatest accuracy is the moment to which the joint that connects the girder to the column is subjected. This moment is the moment at the ends of the girders. If the moments of inertia of the sections are not made proportional to the bending moment determined by method I, the method will not be as accurate as Table 28 indicates.

The moments of inertia of the girder sections of Table 29 are proportional to the bending moments in the respective girders as determined by method II. The moments of inertia of the girder sections of Table 30 are proportional to the bending moments in the respective girders determined by method III. It is apparent from Tables 29-30 and Tables 23-26 that methods II and III are so inaccurate that they should never be used.

The moments of inertia of the girder sections of Table 31 are proportional to the bending moments in the respective girders as determined by method IV. For bents having equal spans and equal column sections this method gives quite accurate results.

While methods I and IV may be quite accurate in some cases, their accuracy depends upon the proportions of the bent. For example, by comparing Tables 27 and 28 it is apparent that if the moments of inertia of the girder sections are not proportional to the bending moments determined by method I, that method will not be as accurate as when the girder sections are so proportioned. Again, in Tables 28 and 31 the sections of the columns are equal and the moments of inertia of the sections of column *A* and girder *a* are equal; but if these relations do not exist, the methods will not in general be as accurate as Tables 28 and 31 seem to indicate.

Any inaccuracy in methods I and IV due to sudden changes in the members of the bent, are in addition to the inaccuracies shown in Tables 28 and 31.

For the bents shown in Tables 28 to 31, the moments determined by the proposed approximate method are exactly the same as the moments determined by the slope-deflection method.

X. Test of a Celluloid Model of a Bent.

23. *Description of Tests.*—In order to check the deflections and the changes in the slopes calculated by the slope-deflection method, a celluloid model of a bent was subjected to known shears, and the resulting deflections and changes in slopes were measured and compared with the calculated values. The model was made of celluloid 1/8 inch thick, and had the general dimensions shown in Fig. 14. A cord passing over a pulley and attached to a weight at one end and to the top of the model at the other, produced a uniform shear in all stories. Paper arms were fastened to the model at points where the members intersect. The movement of these arms indicated the changes in the slopes. One of these arms, A_2D, is shown in Fig. 14. The external force caused the point A_2 to move in approximately a horizontal line and at the same time to turn through a small angle, θ. The paper arm A_2D has the same motion as the point A_2; thus the vertical displacement of D measures the angle θ. The horizontal deflection of the model was obtained by measuring the displacement of a point at the middle of a girder. Paper arms were attached to all four columns at the tops of all stories simultaneously, and the changes in the slopes at all intersections were measured for each application of the load. Similarly, the horizontal deflection was measured at the middle of each of the three girders at the top of each story.

In the original model, known as model No. 1, members No. 1, 2, 3, and 4 were 1/2 inch wide; all other members were 1/4 inch wide. After this model had been tested, member No. 1 was reduced to 1/4 inch; and then the resulting model, now known as No. 2, was tested. Members No. 2, 3, and 4 were successively reduced to 1/4 inch; and the resulting models, now known as models No. 3, 4, and 5, were also tested.

In testing model No. 1, observations were made when loads of 2 1/2 lb., 5 lb., 7 1/2 lb., and 10 lb. were applied. The readings for the two sides of the model agreed very closely in the bottom three stories;

but in the fourth story they did not agree. Thinking that this discrepancy might be due to excessive stresses, when testing model No. 2, a maximum load of only 7½ lb. was used; but there was still the same discrepancy in the readings. Thinking that the apparatus might be out of order, the loads were removed and then applied a second time but the second readings agreed very closely with the first. In testing models No. 3, 4, and 5, loads of 1½ lb., 3 lb., and 4½ lb. were applied. The loads were then removed and applied in the opposite direction. The readings for the two sides of the model continued to agree very closely in the bottom three stories; and in the fourth story they continued to

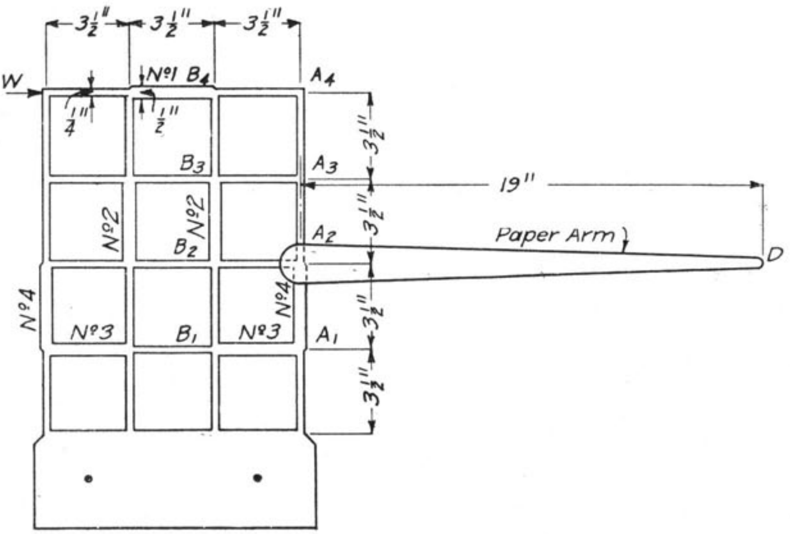

FIG. 14. CELLULOID MODEL.

disagree. It is therefore probable that either the material was not homogeneous or that there were internal stresses or local weaknesses in the upper part of the model.

The observations made in the test of model No. 4 are given in Table 32.

24. *Results of Tests.*—The results of the tests of the models are given in Fig. 15. The models are shown by the sketches at the left of the figure. In these sketches members represented by heavy lines are ½ inch wide, and all other members are ¼ inch wide. The upper diagram of Fig. 15 shows the changes in the slopes and the lower diagram shows the deflections.

In the upper diagram the first group of lines at the left represents the changes in the slope at the point A_1, the second group represents the changes at A_2, and similarly the third and fourth groups represent the changes at A_3 and A_4. The fifth group of lines from the left represents the changes in the slope at the point B_1, the sixth group at B_2, and similarly for the other groups. In the left-hand group of lines the change in the slope at the point A_1 of model No. 1 is laid off from the origin on a horizontal line opposite the sketch of model No. 1; the change in the slope at A_1 of model No. 2 is laid off from the origin on a horizontal line opposite model No. 2; the change in the slope at A_1 of model No. 3 is laid off from the origin on the horizontal line opposite model No. 3; and the change in the slope at A_1 of model No. 4 is laid off from the origin on the horizontal line opposite model No. 4. The full lines connect points which represent the quantities as measured; and the dotted lines connect points which represent the same quantities as computed by the slope-deflection method. The changes in the slopes at A_2, A_3, A_4, B_1, B_2, B_3, and B_4 are shown in a similar manner.

The lower diagram shows the ratios of the deflection to the story height in the different stories of the models. The method of representing the ratios of deflection to story height is similar to the method used to represent the changes in the slopes in the upper diagram.

It will be noticed that the calculated quantities are in general greater than the observed quantities. The reason for this is: In the computations, the length of a member was taken equal to the distance between center lines, whereas the length that is actually free to bend is the distance from outside to outside of the members. This accounts for the difference between the observed and the computed values.

Fig. 15 shows that the changes in the slopes and the ratios of the deflection to the story height as observed and as computed agree closely. In other words, the tests support the theory upon which the slope-deflection method is based.

XI. Discussion of the Assumptions.

25. *Preliminary.*—In making the analysis of the stresses the writers made certain assumptions in regard to the action of the frame when stressed. The effect of inaccuracies in these assumptions will now be considered.

If all of the columns of a story are taken together as a free body, the algebraic sum of the moments at the tops and bottoms of all the columns is equal to the shear on the story multiplied by the story height. As the moments in the columns are balanced by the moments in the

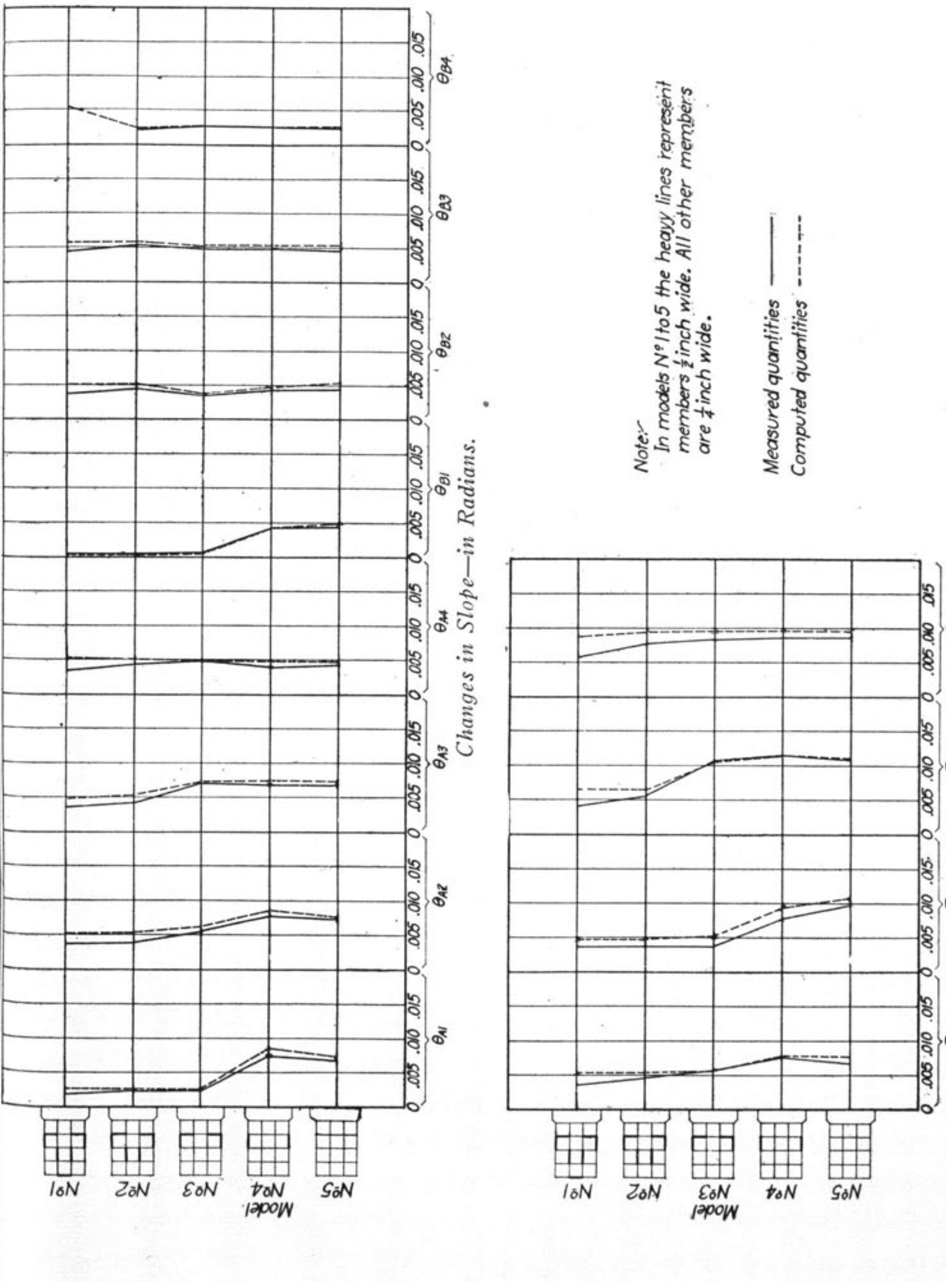

FIG. 15. DIAGRAM OF RESULTS OF THE TESTS OF CELLULOID MODEL.

girders, the algebraic sum of the moments at the ends of the girders is dependent upon the product of the shear in the story and the story height. These facts should be borne in mind in discussing the effect of inaccuracies in the assumptions upon the moments in a bent.

26. *Assumption of Perfect Rigidity.*—According to assumption No. 1, the connections between the columns and girders are perfectly rigid. The truth of this assumption can be determined only by tests. As far as the writers are aware no such tests have been made. While it is undesirable to make a mathematical analysis which is based upon an unverified assumption, some assumption relative to the rigidity of the connections must be made before the stresses in a frame can be determined. The distribution of the stresses depends more upon the relative stiffness than upon the actual stiffness of the connections. In view of these facts, it seems that the assumption of perfect rigidity of the joints is the most logical one that can be made.

27. *Assumption of the Unchanged Length.*—According to assumption No. 2, the change in the length of a member due to the direct stress is equal to zero.

If columns A and B change in length, the moment at the right-hand end of girder a will be given by the equation

$$M_{AB} = 2EK_a\,(2\theta_A + \theta_B - 3R_a),$$

in which R_a is equal to the difference between the changes in the lengths of columns A and B divided by the length of girder a. In the derivation of the general equations in Section VI, R_a was assumed to be equal to zero. If the changes in the lengths of the columns do not alter the values of θ_A and θ_B, the change in the moment ΔM_{AB}, at the right-hand end of girder a due to the changes in the lengths of the columns is:

$$\Delta M_{AB} = 2EK_a\,(-3R_a).$$

The change in the moment at the left-hand end of girder a is equal to the change at the right-hand end. The difference between the changes in length of columns A and B, or the deflection of one end of girder a relative to the other end, is given by the equation:

$d = \dfrac{Pl}{AE}$, in which

$d =$ the deflection,

$P =$ the difference between the stress in column A and the stress in column B,

$l =$ the length of the columns,

$A =$ the area of the column section,

$E =$ the modulus of elasticity of steel.

From Table 19, page 75, the compressive stresses in the first story of columns A and B are 14,464 lb. and 4,587 lb. respectively.

Therefore $P = 14{,}464 - 4{,}587 = 9{,}877$ lb. From Table 11, page 48, $l = 264$ in., $K_a = 30.5$ in.^{3}, and $A = 112.8$ sq. in. $E = 29{,}000{,}000$ lb. per sq. in. Substituting these values in the above equation for d, gives:

$$d = .000798 \text{ in.}$$

By definition, $R_a = \dfrac{d}{l}$, therefore

$$R_a = \frac{.000798}{264} = .00000303, \text{ and}$$

$$\Delta M_{AB} = 2 \times 29{,}000{,}000 \times 30.5\ (-3 \times .00000303) = -16{,}050 \text{ in. lb.}$$

This change in the moment is 5.6 per cent of the moment at the right-hand end of girder a as given in Table 18, page 74. Likewise the moment at the left-hand end of girder a is decreased by 16,050 in. lb., which is 6.6 per cent of the moment as given in Table 18.

The moment at the end of girder b is affected by the changes in the lengths of two columns, B, one on each side of the center line of the bent. One column is subjected to a tensile stress of 4,587 lb., and the other to a compressive stress of 4,587 lb. The deflection of girder b is given by the equation

$$d = \frac{2 \times 4{,}587 \times 264}{112.8 \times 29{,}000{,}000} = .00074 \text{ inches.}$$

$$R_b = \frac{.00074}{216} = .00000343.$$

$$\Delta M_{BB} = 2 \times 29{,}000{,}000 \times 37.3 \times 3 \times .00000343 = 22{,}300 \text{ in. lb.,}$$

which is 9.0 per cent of the moment at the end of girder b as given in Table 18.

The change in the length of a column is a function of the unit direct stress and also a function of the story height. The first story of the building considered is much higher (22 ft.) than the other stories, and the unit direct stress in the columns in the first story is greater than in the stories above; hence the change in the length of the columns is very much less in the other stories than in the first story. Therefore the changes in the moments for the other stories will be less than those in the first story computed above.

The changes in the moments in the columns have been determined and it remains to consider the effect of the direct stresses in the girders. The direct stress in a girder is very small in comparison with the direct stress in a column and therefore may be neglected.

If the changes in the slopes were not affected by the changes in the lengths of the columns, the moments at the ends of girders *a* and *b* would be decreased. As the sum of these moments is determined by the product of the shear in the story and the story height, they can not all be decreased simultaneously. Therefore all of the changes in the slopes must be increased until the sum of the moments at the ends of the girders will balance the moments in the columns. This will make the moments at the ends of the girders practically the same as they would have been if the columns had not changed in length. If the moments at the ends of the girders are not materially affected, the moments at the ends of the columns will not be materially affected. Therefore although the direct stresses in the columns do change the lengths of the columns, they do not affect the stresses in the frame to any great extent.

28. *Assumption as to Length of Members.*—In accordance with assumption No. 3, the length of a member was taken as the distance between the center lines of the members which it intersects. This makes the changes in the slopes and deflections, as calculated, greater than the actual values. The effect of the inaccuracy of this assumption upon the distribution of the moments is, however, not so apparent.

The curves in Fig. 6, 7, and 8 show that the distribution of the moment in a story depends upon the relative values of K of the members, but that it takes a comparatively large change in the K of a member to appreciably affect the distribution of the moments. The fact that the length of a member has been taken equal to the distance between center lines, has but little effect upon the relative values of K for the members; and therefore does not materially affect the distribution of the moments.

29. *Assumption as to Deflection Due to Shear.*—According to assumption No. 4, the internal shearing stresses in a member produce no deflection. The distribution of the stresses is dependent, not upon the actual deflection of the columns due to shear, but upon the differences between the deflections in the different columns. The shears on the columns are small and the differences between the shears are still smaller; and therefore the assumption that the deflection due to shear is equal to zero will not cause any appreciable error.

30. *Assumption as to Load.*—According to assumption No. 5, the

entire wind load is resisted by the steel frame. The walls of a building no doubt help to resist the wind loads, but the resistance which they exert is uncertain. If a portion of the load is considered as being resisted by the walls the stresses in the steel frame are correspondingly reduced; but the method of determining the stresses is not affected.

XII. Conclusions.

As a result of the investigation described in this bulletin the following conclusions can be made relative to the methods used to determine the wind stresses in the steel frames of office buildings.

a. Methods II and III of Section II are so inaccurate that they should never be used; I and IV are quite accurate in some cases, but they may give results which are seriously in error.

b. The method presented in Sections VI and VII, and designated as the slope-deflection method, contains no approximations except those in the assumptions. It has been shown that the inaccuracies in the assumptions do not materially affect the results. Therefore the method is very accurate.

c. While the slope-deflection method is long, it could be used in the actual design of a building; but it has its greatest value as a standard by means of which the accuracy of the approximate methods may be determined.

d. The proposed approximate method is short; and, except at points where there are very large changes in the size of the members, gives results which are accurate enough to be used in the actual design of a building.

Table 11.

Properties of the Columns and Girders in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

	Story Number	Section of Member			Area of Section Sq. In.	Moment of Inertia—(Inches)^4					Length of Columns $= h$ of Girders $= l$ Inches	K for Columns $= \frac{I}{h}$ for Girders $= \frac{I}{l}$ (Inches)^3
		Web Plate	4 Angles	Cover Plate		Web	4 Angles Primary	4 Angles Secondary	Cover Plate	Total		
Column A	1	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x1$\frac{1}{4}$	112.80	358	318	2190	3950	6816	264	25.8
	2	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x1$\frac{1}{4}$	112.80	358	318	2190	3950	6816	192	35.6
	3 and 4	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x1	103.80	358	318	2190	3080	5946	168	35.4
	5 and 6	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x$\frac{11}{16}$	92.55	358	318	2190	2060	4926	168	29.4
	7 and 8	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x$\frac{7}{16}$	83.55	358	318	2190	1266	4132	144	28.7
	9 and 10	17x$\frac{15}{16}$	8x8x$\frac{15}{16}$		72.42	384	337	2315		3036	144	21.1
	11 and 12	17x$\frac{7}{8}$	8x8x$\frac{13}{16}$		64.24	358	299	2050		2707	144	18.8
	13 and 14	17x$\frac{3}{4}$	8x6x$\frac{13}{16}$		53.79	308	131	2195		2634	144	18.3
	15 and 16	17x$\frac{1}{2}$	8x6x$\frac{5}{8}$		42.26	205	105	1745		2055	144	14.3
	17 and above	17x$\frac{1}{2}$	8x6x$\frac{9}{16}$		38.86	205	96	1590		1891	144	13.1
Column B	1	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x1$\frac{1}{4}$	112.80	358	318	2190	3950	6816	264	25.8
	2	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x1$\frac{1}{4}$	112.80	358	318	2190	3950	6816	192	35.6
	3 and 4	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x1	103.80	358	318	2190	3080	5946	168	35.5
	5 and 6	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x$\frac{3}{4}$	94.80	358	318	2190	2240	5106	168	30.4
	7 and 8	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x$\frac{1}{2}$	85.80	358	318	2190	1459	4325	144	30.0
	9 and 10	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x$\frac{5}{16}$	79.05	358	318	2190	892	3758	144	26.1
	11 and 12	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$		67.80	358	318	2190		2866	144	19.9
	13 and 14	17x$\frac{7}{8}$	8x6x$\frac{13}{16}$		57.92	358	131	2195		2684	144	18.6
	15 and 16	17x$\frac{5}{8}$	8x6x$\frac{5}{8}$		44.39	256	105	1745		2106	144	14.6
	17 and above	17x$\frac{5}{8}$	8x6x$\frac{9}{16}$		41.00	256	96	1544		1896	144	13.2
Girders in Bay a	1	42x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{3}{8}$		29.43	2315	13	5730		8058	264	30.5
	2 and 3	36x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{3}{8}$		27.18	1458	13	4170		5641	264	21.4
	4 and 5	36x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		25.70	1458	13	3690		5161	264	19.5
	6	30x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{3}{8}$		24.93	844	13	2860		3717	264	14.1
	7	30x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		23.45	844	13	2530		3387	264	12.8
	8 and above	24x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		21.20	432	13	1580		2025	264	7.7
Girders in Bay b	1	42x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{3}{8}$		29.43	2315	13	5730		8058	216	37.3
	2	36x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{7}{16}$		29.38	1458	15	4830		6303	216	29.2
	3 and 4	36x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{3}{8}$		27.18	1458	13	4170		5641	216	26.2
	5	36x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		25.70	1458	13	3690		5161	216	23.8
	6	30x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{3}{8}$		24.93	844	13	2860		3717	216	17.2
	7	30x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		23.45	844	13	2530		3387	216	15.7
	8 and above	24x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		21.20	432	13	1580		2025	216	9.4

Table 12.

Numerical Values of the Constants in the Equations of Table 3 for the Symmetrical Three-span Twenty-story Bent shown in Fig. 5.

All quantities are expressed in inches³

Story No.	Values of $K\left(=\frac{I}{h}\right)$ for		Values of $K\left(=\frac{I}{l}\right)$ for		Values of $J\left[=2\Sigma\left(\frac{I}{h}+\frac{I}{l}\right)\right]$ for all members intersecting at the		Values of $N\ (=2\sum K)$ for all Columns in a Story
	Column A	Column B	Girders in Bay a at Top of Story	Girders in Bay b at Top of Story	Top of Column A	Top of Column B	
1	25.8	25.8	30.5	37.3	183.8	258.4	206.8
2	35.6	35.6	21.4	29.2	184.8	243.4	284.8
3	35.4	35.5	21.4	26.2	184.4	237.2	283.6
4	35.4	35.5	19.5	26.2	168.6	223.2	283.6
5	29.4	30.4	19.5	23.8	156.6	208.2	239.2
6	29.4	30.4	14.1	17.2	144.4	183.4	239.2
7	28.7	30.0	12.8	15.7	140.4	177.0	234.8
8	28.7	30.0	7.7	9.4	115.0	146.4	234.8
9	21.1	26.1	7.7	9.4	99.8	138.6	188.8
10	21.1	26.1	7.7	9.4	95.2	126.2	188.8
11	18.8	19.9	7.7	9.4	90.6	113.8	154.8
12	18.8	19.9	7.7	9.4	89.6	111.2	154.8
13	18.3	18.6	7.7	9.4	88.6	108.6	147.6
14	18.3	18.6	7.7	9.4	80.6	100.6	147.6
15	14.3	14.6	7.7	9.4	72.6	92.6	115.6
16	14.3	14.6	7.7	9.4	70.2	89.8	115.6
17	13.1	13.2	7.7	9.4	67.8	87.0	105.2
18	13.1	13.2	7.7	9.4	67.8	87.0	105.2
19	13.1	13.2	7.7	9.4	67.8	87.0	105.2
20	13.1	13.2	7.7	9.4	41.6	60.6	105.2

TABLE 13.

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3			Story No. 4			Story No. 5			
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	θ_{A3}	θ_{B3}	R_4	θ_{A4}	θ_{B4}	R_5	θ_{A5}	θ_{B5}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															
A	—206.8	51.6	51.6													—117.2
B	— 77.4	183.8	30.5	—106.8	35.6											0.0
C	— 77.4	30.5	295.7	—106.8		35.6										0.0
D		71.2	71.2	—284.8	71.2	71.2										—78.9
E		35.6		—106.8	184.8	21.4	—106.2	35.4								0.0
F			35.6	—106.8	21.4	272.6	—106.5		35.5							0.0
G					70.8	71.0	—283.6	70.8	71.0							—64.5
H					35.4		—106.2	184.4	21.4	—106.2	35.4					0.0
I						35.5	—106.5	21.4	263.4	—106.5		35.5				0.0
J								70.8	71.0	—283.6	70.8	71.0				—60.5
K								35.4		—106.2	168.6	19.5	—88.2	29.4		0.0
L									35.5	—106.5	19.5	249.4	—91.2		30.4	0.0

TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 4			Story No. 5			Story No. 6			Story No. 7			Story No. 8			
	R_4	θ_{A4}	θ_{B4}	R_5	θ_{A5}	θ_{B5}	R_6	θ_{A6}	θ_{B6}	R_7	θ_{A7}	θ_{B7}	R_8	θ_{A8}	θ_{B8}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															
M		58.8	60.8	—239.2	58.8	60.8										—56.5
N		29.4		— 88.2	156.6	19.5	— 88.2	29.4								0.0
O			30.4	— 91.2	19.5	232.0	— 91.2		—30.4							0.0
P					58.8	60.8	—239.2	58.8	60.8							—52.5
Q					29.4		— 88.2	144.4	14.1	— 86.1	28.7					0.0
R						30.4	— 91.2	14.1	200.6	— 90.0		30.0				0.0
S								57.4	60.0	—234.8	57.4	60.0				—41.8
T								28.7		— 86.1	140.4	12.8	—86.1	28.7		0.0
U									30.0	— 90.0	12.8	192.7	—90.0		30.0	0.0

TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 7			Story No. 8			Story No. 9			Story No. 10			Story No. 11			
	R_7	θ_{A7}	θ_{B7}	R_8	θ_{A8}	θ_{B8}	R_9	θ_{A9}	θ_{B9}	R_{10}	θ_{A10}	θ_{B10}	R_{11}	θ_{A11}	θ_{B11}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															Coefficient of .0001
V		57.4	60.0	—234.8	57.4	60.0										—38.7
W		28.7		— 86.1	115.0	7.7	— 63.3	21.1								0.0
X			30.0	— 90.0	7.7	155.8	— 78.3		26.1							0.0
Y					42.2	52.2	—188.8	42.2	52.2							—35.8
Z					21.1		— 63.3	99.8	7.7	— 63.3	21.1					0.0
a						26.1	— 78.3	7.7	148.0	— 78.3		26.1				0.0
b								42.2	52.2	—188.8	42.2	52.2				—32.8
c								21.1		— 63.3	95.2	7.7	—56.4	18.8		0.0
d									26.1	— 78.3	7.7	135.6	—59.7		19.9	0.0

TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 10			Story No. 11			Story No. 12			Story No. 13			Story No. 14			
	R_{10}	θ_{A10}	θ_{B10}	R_{11}	θ_{A11}	θ_{B11}	R_{12}	θ_{A12}	θ_{B12}	R_{13}	θ_{A13}	θ_{B13}	R_{14}	θ_{A14}	θ_{B14}	Co-efficient of .0001
	Coefficient of Unknown Slopes and Ratios of Deflection to Story Height															
e		37.6	39.8	—154.8	37.6	39.8										—29.8
f		18.8		— 56.4	90.6	7.7	— 56.4	18.8								0.0
g			19.9	— 59.7	7.7	123.2	— 59.7		19.9							0.0
h					37.6	39.8	—154.8	37.6	39.8							—26.8
i					18.8		— 56.4	89.6	7.7	— 54.9	18.3					0.0
j						19.9	— 59.7	7.7	120.6	— 55.8		18.6				0.0
k										—147.6	36.6	37.2				—23.8
l										— 54.9	88.6	7.7	—54.9	18.3		0.0
m										— 55.8	7.7	118.0	—55.8		18.6	0.0

TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 13			Story No. 14			Story No. 15			Story No. 16			Story No. 17			
	R_{13}	θ_{A13}	θ_{B13}	R_{14}	θ_{A14}	θ_{B14}	R_{15}	θ_{A15}	θ_{B15}	R_{16}	θ_{A16}	θ_{B16}	R_{17}	θ_{A17}	θ_{B17}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															Co-efficient of .0001
n		36.6	37.2	—147.6	36.6	37.2										—20.9
o		18.3		— 54.9	80.6	7.7	— 42.9	14.3								0.0
p			18.6	— 55.8	7.7	110.0	— 43.8		14.6							0.0
q					28.6	29.2	—115.6	28.6	29.2							—17.9
r					14.3		— 42.9	72.6	7.7	— 42.9	14.3					0.0
s						14.6	— 43.8	7.7	102.0	— 43.8		14.6				0.0
t								28.6	29.2	—115.6	28.6	29.2				—14.9
u								14.3		— 42.9	70.2	7.7	—39.3	13.1		0.0
v									14.6	— 43.8	7.7	99.2	—39.6		13.2	0.0

TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 16			Story No. 17			Story No. 18			Story No. 19			Story No. 20			
	R_{16}	θ_{A16}	θ_{B16}	R_{17}	θ_{A17}	θ_{B17}	R_{18}	θ_{A18}	θ_{B18}	R_{19}	θ_{A19}	θ_{B19}	R_{20}	θ_{A20}	θ_{B20}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															Co-efficient of .0001
w		26.2	26.4	—105.2	26.2	26.4										—11.9
x		13.1		— 39.3	67.8	7.7	— 39.3	13.1								0.0
y			13.2	— 39.6	7.7	96.4	— 39.6		13.2							0.0
z					26.2	26.4	—105.2	26.2	26.4							—8.94
A′					13.1		— 39.3	67.8	7.7	— 39.3	13.1					0.0
B′						13.2	— 39.6	7.7	96.4	— 39.6		13.2				0.0
C′								26.2	26.4	—105.2	26.2	26.4				—5.96
D′								13.1		— 39.3	67.8	7.7	— 39.3	13.1		0.0
E′									13.2	— 39.6	7.7	96.4	— 39.6		13.2	0.0
F′											26.2	26.4	—105.2	26.2	26.4	—2.98
G′											13.1		— 39.3	41.6	7.7	0.0
H′												13.2	— 39.6	7.7	70.0	0.0

TABLE 14.

ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3			Story No. 4			
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	θ_{A3}	θ_{B3}	R_4	θ_{A4}	θ_{B4}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Co-efficient of .0001
A	1.0000	—0.2495	—0.2495										0.5667
B	1.0000	—2.3747	—0.3940	1.3798	—0.4599								0.0000
C	1.0000	—0.3940	—3.8204	1.3798		—0.4599							0.0000
A–B	0	2.1252	0.1445	—1.3798	0.4599								0.5667
B–C	0	—1.9806	3.4263	0	—0.4599	0.4599							0.0000
1		1.0000	0.0680	—0.6494	0.2164								0.2667
2		1.0000	—1.7300		0.2322	—0.2322							0.0000
D		1.0000	1.0000	—4.0000	1.0000	1.0000							—1.1082
E		1.0000		—3.0000	5.1912	0.6012	—2.9834	0.9944					0.0000
1–2		0	1.7980	—0.6494	—0.0158	0.2322							0.2667
2–D		0	—2.7300	4.0000	0.7678	—1.2322							1.1082
D–E		0	1.0000	—1.0000	—4.1912	0.3988	2.9834	—0.9944					—1.1082
3			1	—0.3611	—0.0088	0.1292							0.1483
4			1	—1.4652	0.2812	0.4513							—0.4059
5			1	—1.0000	—4.1912	0.3988	2.9834	—0.9941					—1.1082
F			1	—3.0000	0.6012	7.6575	—2.9915		0.9972				0.0000
3–4			0	1.1040	—0.2900	—0.3222							0.5543
4–5			0	—0.4652	4.4724	0.0525	—2.9834	0.9944					0.7023
5–F			0	2.0000	—4.7924	—7.2587	5.9749	—0.9944	—0.9972				—1.1082
6				1	—0.2627	—0.2918							0.5021
7				1	—9.6140	—0.1128	6.4127	—2.1378					—1.5095
8				1	—2.3962	—3.6293	2.9874	—0.4972	—0.4986				—0.5541
6–7				0	9.3513	—0.1790	—6.4127	2.1378					2.0116
7–8				0	—7.2178	3.5165	3.4252	—1.6406	0.4986				—0.9554
9					1	—0.0191	—0.6857	0.2286					0.2151
10					1	—0.4872	—0.4746	0.2273	—0.0691				0.1324
G					1	1.0028	—4.0055	1.0000	1.0028				—0.9110
H					1		—3.0000	5.2091	0.6045	—3.0000	1.0000		0.0000
9–10					0	0.4681	—0.2111	0.0013	0.0691				0.0827
10–G					0	—1.4900	3.5309	—0.7727	—1.0719				1.0434
G–H					0	1.0028	—1.0055	—4.2091	0.3983	3.0000	—1.0000		—0.9110
11						1	—0.4511	0.0027	0.1476				0.1768
12						1	—2.3700	0.5186	0.7194				—0.6990
13						1	—1.0028	—4.1971	0.3971	2.9915	—0.9972		—0.9084
I						1	—3.0000	0.6028	7.4200	—3.0000		1.0000	0.0000
11–12						0	1.0189	—0.5159	—0.5718				0.8758
12–13						0	—1.3672	4.7157	0.3223	—2.9915	0.9972		0.2094
13–I						0	1.9972	—4.7999	—7.0230	5.9915	—0.9972	—1.0000	—0.9084

Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 3			Story No. 4			Story No. 5			Story No. 6			
	R_3	θ_{A3}	θ_{B3}	R_4	θ_{A4}	θ_{B4}	R_5	θ_{A5}	θ_{B5}	R_6	θ_{A6}	θ_{B6}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												
14	1	-0.2689	-0.2980										0.4564
15	1	-3.4490	-0.2357	2.1881	-0.7294								-0.1532
16	1	-2.4033	-3.5179	2.9999	-0.4993	-0.5007							-0.4548
14-15	0	3.1802	-0.0623	-2.1881	0.7294								0.6096
14-16	0	2.1344	3.2199	-2.9999	0.4993	0.5007							0.9112
17		1	-0.0196	-0.6881	0.2294								0.1917
18		1	1.5086	-1.4057	0.2339	0.2346							0.4270
J		1	1.0028	-4.0057	1.0000	1.0028							-0.8546
K		1		-3.0000	4.7628	0.5508	-2.4914	0.8302					0.0000
17-18		0	-1.5282	0.7176	-0.0046	-0.2346							-0.2353
18-K		0	1.5086	1.5943	-4.5290	-0.3162	2.4914	-0.8302					0.4270
J-K		0	1.0028	-1.0057	-3.7628	0.4520	2.4914	-0.8302					-0.8546
19			1	-0.4696	0.0030	0.1535							0.1540
20			1	1.0568	-3.0019	-0.2096	1.6514	-0.5503					0 2830
21			1	-1.0028	-3.7520	0.4507	2.4842	-0.8279					-0.8521
L			1	-3.0000	0.5496	7.0255	-2.5691		0.8564				0.0000
19-20			0	-1.5264	3.0049	0.3631	-1.6514	0.5503					-0.1290
20-21			0	2.0596	0.7501	-0.6603	-0.8328	0.2776					1.1352
20-L			0	4.0568	-3.5515	-7.2351	4.2205	-0.5503	-0.8564				0.2830
22				1	-1.9687	-0.2379	1.0819	-0.3605					0.0846
23				1	0.3642	-0.3206	-0.4044	0.1348					0.5512
24				1	-0.8754	-1.7836	1.0404	-0.1357	-0.2111				0.0697
22-23				0	-2.3329	0.0827	1.4863	-0.4953					-0.4666
23-24				0	1.2397	1.4630	-1.4448	0.2705	0.2111				0.4815
25					1	-0.0355	-0.6371	0.2123					0.2000
26					1	1.1803	-1.1657	0.2182	0.1703				0.3884
M					1	1.0340	-4.0684	1.0000	1.0340				-0.9609
N					1		-3.0000	5.3260	0.6632	-3.0000	1.0000		0.0000
25-26					0	-1.2158	0.5286	-0.0059	-0.1703				-0.1884
25-M					0	-1.0695	3.4313	-0.7877	-1.0340				1.1609
M-N					0	1.0340	-1.0684	-4.3260	0.3708	3.0000	-1.0000		-0.9609
27						1	-0.4348	0.0048	0.1401				0.1550
28						1	-3.2083	0.7365	0.9668				-1.0855
29						1	-1.0333	-4.1842	0.3586	2.9016	-0.9672		-0.9294
O						1	-3.0000	-0.6415	7.6321	-3.0000		1.0000	0.0000
27-28						0	2.7735	-0.7316	-0.8267				1.2405
28-29						0	-2.1750	4.9207	0.6082	-2.9016	0.9672		-0.1561
29-O						0	1.9667	-4.8257	-7.2735	5.9016	-0.9672	-1.0000	-0.9294
30							1	-0.2638	-0.2981				0.4473
31							1	-2.2623	-0.2796	1.3341	-0.4447		0.0718
32							1	-2.4540	-3.6986	3.0004	-0.4918	-0.5085	-0.4726
30-31							0	1.9985	-0.0185	-1.3341	0.4447		0.3755
30-32							0	2.1902	3.4005	-3.0004	0.4918	0.5085	0.9199

TABLE 14.—(Continued).

ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation.												Right-Hand Member of Equation
	Story No. 5			Story No. 6			Story No. 7			Story No. 8			
	R_5	θ_{A5}	θ_{B5}	R_6	θ_{A6}	θ_{B6}	R_7	θ_{A7}	θ_{B7}	R_8	θ_{A8}	θ_{B8}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height.												
33		1	−0.0092	−0.6676	0.2225								0.1879
34		1	1.5526	−1.3698	0.2246	0.2322							0.4200
P		1	1.0341	−4.0682	1.0000	1.0341							−0.8929
Q		1		−3.0000	4.9120	0.4796	−2.9285	0.9762					0.0000
33–34		0	−1.5618	0.7022	−0.0020	−0.2322							−0.2321
33–P		0	−1.0433	3.4006	−0.7775	−1.0341							1.0808
P–Q		0	1.0341	−1.0682	−3.9120	0.5545	2.9285	−0.9762					−0.8929
35			1	−0.4497	0.0013	0.1487							0.1486
36			1	−3.2593	0.7452	0.9911							−1.0359
37			1	−1.0330	−3.7818	0.5362	2.8320	−0.9440					−0.8634
R			1	−3.0000	0.4638	6.5993	−2.9606		0.9870				0.0000
35–36			0	2.8097	−0.7439	−0.8425							1.1845
36–37			0	−2.2263	4.5270	0.4549	−2.8320	0.9440					−0.1724
37–R			0	1.9670	−4.2456	−6.0631	5.7926	−0.9440	−0.9870				−0.8634
38				1	−0.2648	−0.2998							0.4216
39				1	−2.0335	−0.2044	1.2721	−0.4240					0.0775
40				1	−2.1586	−3.0827	2.9451	−0.4799	−0.5018				−0.4390
38–39				0	1.7687	−0.0955	−1.2721	0.4240					0.3441
38–40				0	1.8938	2.7828	−2.9451	0.4799	0.5018				0.8606
41					1	−0.0540	−0.7192	0.2397					0.1946
42					1	1.4695	−1.5553	0.2534	0.2650				0.4544
S					1	1.0453	−4.0909	1.0000	1.0453				−0.7281
T					1		−3.0000	4.8920	0.4460	−3.0000	1.0000		0.0000
41–42					0	−1.5235	0.8361	−0.0137	−0.2650				−0.2599
41–S					0	−1.0993	3.3717	−0.7603	−1.0453				0.9227
S–T					0	1.0453	−1.0909	−3.8920	0.5993	3.0000	−1.0000		−0.7281
43						1	−0.5489	0.0090	0.1739				0.1706
44						1	−3.0672	0.6916	0.9510				−0.8394
45						1	−1.0436	−3.7234	0.5733	2.8700	−0.9567		0.6964
U						1	−3.0000	0.4267	6.4233	−3.0000		1.0000	0.0000
43–44						0	2.5183	−0.6826	−0.7781				1.0100
44–45						0	−2.0236	4.4150	0.3777	−2.8700	0.9567		−0.1430
45–U						0	1.9564	−4.1501	−5.8500	5.8700	−0.9567	−1.0000	−0.6964
46							1	−0.2711	−0.3090				0.4011
47							1	−2.1818	−0.1867	1.4184	−0.4728		0.0706
48							1	−2.1213	−2.9903	3.0006	−0.4890	−0.5111	−0.3560
46–47							0	1.9108	−0.1223	−1.4184	0.4728		0.3304
46–48							0	1.8502	2.6813	−3.0006	0.4890	0.5111	0.7570

Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 7			Story No. 8			Story No. 9			Story No. 10			
	R_7	θ_{A7}	θ_{B7}	R_8	θ_{A8}	θ_{B8}	R_9	θ_{A9}	θ_{B9}	R_{10}	θ_{A10}	θ_{B10}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Co-efficient of .0001
49		1	-0.0640	-0.7422	0.2474								0.1729
50		1	1.4491	-1.6219	0.2643	0.2763							0.4093
V		1	1.0453	-4.0909	1.0000	1.0453							-0.6741
W		1		-3.0000	4.0073	0.2683	-2.2056	0.7352					0.0000
49-50		0	-1.5131	0.8797	-0.0169	-0.2763							-0.2363
49-V		0	-1.1093	3.3487	-0.7526	-1.0453							0.8470
V-W		0	1.0453	-1.0909	-3.0073	-0.7770	2.2056	-0.7352					-0.6741
51			1	-0.5814	0.0112	0.1826							0.1562
52			1	-3.0186	0.6784	0.9423							-0.7636
53			1	-1.0436	-2.8769	0.7433	2.1100	-0.7036					-0.6449
X			1	-3.0000	0.2567	5.1933	-2.6100		0.8700				0.0000
51-52			0	2.4372	-0.6672	-0.7598							0.9198
52-53			0	-1.9750	3.5553	0.1990	-2.1100	0.7036					-0.1187
53-X			0	1.9564	-3.1336	-4.4500	4.7200	-0.7036	-0.8700				-0.6449
54				1	-0.2738	-0.3117							0.3774
55				1	-1.8002	-0.1008	1.0685	-0.3563					0.0601
56				1	-1 6019	-2.2747	2.4127	-0.3597	-0.4447				-0.3296
54-55				0	1.5265	-0.2110	-1.0685	0.3563					0.3173
54-56				0	1.3281	1.9630	-2.4127	0.3597	0.4447				0.7070
57					1	-0.1382	-0.7000	0.2334					0.2079
58					1	1.4779	-1.8165	0.2708	0.3348				0.5323
Y					1	1.2371	-4.4743	1.0000	1.2371				-0.8484
Z					1		-3.0000	4.7299	0.3649	-3.0000	1.0000		0.0000
57-58					0	-1.6161	1.1165	-0.0374	-0.3348				-0.3245
57-Y					0	-1.3753	3.7743	-0.7666	-1.2371				1.0562
Y-Z					0	1.2371	-1.4743	-3.7299	0.8722	3.0000	-1.0000		-0.8484
59						1	-0.6908	0.0231	0.2072				0.2008
60						1	-2.7443	0.5574	0.8995				-0.7681
61						1	-1.1918	-3.0151	0.7051	2.4253	-0.8084		-0.6859
a						1	-3.0000	0.2950	5.6705	-3.0000		1.0000	0.0000
59-60						0	2.0535	-0.5342	-0.6924				0.9689
60-61						0	-1.5525	3.5725	0.1945	-2.4253	0.8084		-0.0822
61-a						0	1.8082	-3.3102	-4.9654	5.4253	-0.8084	-1.0000	-0.6859
62							1	-0.2602	-0.3372				0.4719
63							1	-2.3013	-0.1253	1.5624	-0.5207		0.0529
64							1	-1.8308	-2.7462	3.0000	-0.4471	-0.5531	-0.3793
62-63							0	2.0411	-0.2119	-1.5624	0.5207		0.4189
62-64							0	1.5706	2.4090	-3.0000	0.4471	0.5531	0.8512

TABLE 14.—(Continued).

ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 9			Story No. 10			Story No. 11			Story No. 12			
	R_9	θ_{A9}	θ_{B9}	R_{10}	θ_{A10}	θ_{B10}	R_{11}	θ_{A11}	θ_{B11}	R_{12}	θ_{A12}	θ_{B12}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												
65		1	−0.1038	−0.7655	0.2551								0.2053
66		1	1.5340	−1.9103	0.2847	0.3522							0.5420
b		1	1.2371	−4.4743	1.0000	1.2371							−0.7771
c		1		−3.0000	4.5120	0.3649	−2.6730	0.8911					0.0000
65–66		0	−1.6378	1.1448	−0.0296	−0.3522							−0.3367
65–b		0	−1.3409	3.7088	−0.7449	−1.2371							0.9824
b–c		0	1.2371	−1.4743	−3.5120	0.8722	2.6730	−0.8911					−0.7771
67			1	−0.6991	0.0180	0.2151							0.2056
68			1	−2.7660	0.5555	0.9226							−0.7327
69			1	−1.1918	−2.8392	0.7050	2.1610	−0.7204					−0.6283
d			1	−3.0000	0.2950	5.1954	−2.2873		0.7624				0.0000
67–68			0	2.0669	−0.5375	−0.7075							0.9383
68–69			0	−1.5742	3.3947	0.2176	−2.1610	0.7204					−0.1044
69–d			0	1.8082	−3.1342	−4.4904	4.4483	−0.7204	−0.7624				−0.6283
70				1	−0.2601	−0.3423							0.4540
71				1	−2.1566	−0.1382	1.3728	−0.4576					0.0663
72				1	−1.7333	−2.4833	2.4602	−0.3984	−0.4216				−0.3474
70–71				0	1.8865	−0.2041	−1.3728	0.4576					0.3877
70–72				0	1.4732	2.1310	−2.4602	0.3984	0.4216				0.8014
73					1	−0.1082	−0.7277	0.2426					0.2055
74					1	1.4464	−1.6700	0.2705	0.2862				0.5441
e					1	1.0585	−4.1170	1.0000	1.0585				−0.7926
f					1		−3.0000	4.8195	0.4096	−3.0000	1.0000		0.0000
73–74					0	−1.5547	0.9422	−0.0278	−0.2862				−0.3385
73–e					0	−1.1668	3.3893	−0.7574	−1.0585				0.9981
e–f					0	1.0585	−1.1170	−3.8195	0.6499	3.0000	−1.0000		−0.7926
75						1	−0.6061	0.0179	0.1841				0.2178
76						1	−2.9051	0.6492	0.9076				−0.8555
77						1	−1.0553	−3.6083	0.6140	2.8343	−0.9449		−0.7488
g						1	−3.0000	0.3869	6.1913	−3.0000		1.0000	0.0000
75–76						0	2.2990	−0.6313	−0.7235				1.0733
76–77						0	−1.8498	4.2575	0.2936	−2.8343	0.9449		−0.1067
77–g						0	1.9447	−3.9952	−5.5773	5.8343	−0.9449	−1.0000	−0.7488
78							1	−0.2746	−0.3147				0.4669
79							1	−2.3018	−0.1587	1.5323	−0.5109		0.0577
80							1	−2.0547	−2.8682	3.0003	−0.4859	−0.5143	−0.3851
78–79							0	2.0272	−0.1560	−1.5323	0.5109		0.4092
78–80							0	1.7801	2.5535	−3.0000	0.4859	0.5143	0.8519

TABLE 14.—(Continued).

ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 11			Story No. 12			Story No. 13			Story No. 14			
	R_{11}	θ_{A11}	θ_{B11}	R_{12}	θ_{A12}	θ_{B12}	R_{13}	θ_{A13}	θ_{B13}	R_{14}	θ_{A14}	θ_{B14}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												
81		1	−0.0769	−0.7559	0.2520								0.2018
82		1	1.4344	−1.6854	0.2730	0.2889							0.4786
h		1	1.0585	−4.1170	1.0000	1.0585							−0.7128
i		1		−3.0000	4.7660	0.4096	−2.9205	0.9734					0.0000
81-82		0	−1.5113	0.9294	−0.0210	−0.2889							−0.2768
81-h		0	−1.1355	3.3611	−0.7480	−1.0585							0.9147
h-i		0	1.0585	−1.1170	−3.7660	0.6489	2.9205	−0.9734					−0.7128
83			1	−0.6150	0.0139	0.1911							0.1831
84			1	−2.9602	0.6587	0.9323							−0.8055
85			1	−1.0553	−3.5583	0.6131	2.7592	−0.9197					−0.6734
j			1	−3.0000	0.3869	6.0609	−2.8040		0.9347				0.0000
83-84			0	2.3452	−0.6449	−0.7412							0.9886
84-85			0	−1.9049	4.2170	0.3192	−2.7592	0.9197					−0.1321
85-j			0	1.9447	−3.9452	−5.4478	5.5632	−0.9197	−0.9347				−0.6734
86				1	−0.2750	−0.3160							0.4216
87				1	−2.2137	−0.1676	1.4484	−0.4828					0.0693
88				1	−2.0288	−2.8015	2.8609	−0.4730	−0.4807				−0.3463
86-87				0	1.9387	−0.1485	−1.4484	0.4828					0.3522
86-88				0	1.7539	2.4855	−2.8609	0.4730	0.4807				0.7679
89					1	−0.0766	−0.7471	0.2490					0.1816
90					1	1.4173	−1.6314	0.2697	0.2741				0.4378
k					1	1.0164	−4.0329	1.0000	1.0164				−0.6503
l					1		−3.0000	4.8417	0.4208	−3.0000	1.0000		0.0000
89-90					0	−1.4939	0.8842	−0.0207	−0.2741				−0.2562
89-k					0	−1.0930	3.2857	−0.7510	−1.0164				0.8320
k-l					0	1.0164	−1.0329	−3.8417	0.5956	3.0000	−1.0000		−0.6503
91						1	−0.5920	0.0138	0.1835				0.1715
92						1	−3.0063	0.6871	0.9299				−0.7612
93						1	−1.0163	−3.7802	0.5861	2.9520	−0.9840		−0.6399
m						1	−3.0000	0.4140	6.3443	−3.0000		1.0000	0.0000
91-92						0	2.4143	−0.6733	−0.7464				0.9327
92-93						0	−1.9900	4.4673	0.3438	−2.9520	0.9840		−0.1213
93-m						0	1.9837	−4.1942	−5.7582	5.9520	−0.9840	−1.0000	−0.6399
94							1	−0.2789	−0.3092				0.3863
95							1	−2.2413	−0.1728	1.4834	−0.4945		0.0610
96							1	−2.1143	−2.9030	3.0000	−0.4961	−0.5042	−0.3226
94-95							0	1.9624	−0.1364	−1.4834	0.4945		0.3254
94-96							0	1.8354	2.5938	−3.0000	0.4961	0.5042	0.7089

TABLE 14.—(Continued).

ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 13			Story No. 14			Story No. 15			Story No. 16			
	R_{13}	θ_{A13}	θ_{B13}	R_{14}	θ_{A14}	θ_{B14}	R_{15}	θ_{A15}	θ_{B15}	R_{16}	θ_{A16}	θ_{B16}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Co-efficient of .0001
97		1	—0.0695	—0.7559	0.2520								0.1658
98		1	1.4135	—1.6349	0.2704	0.2747							0.3863
n		1	1.0164	—4.0329	1.0000	1.0164							—0.5711
o		1		—3.0000	4.4044	0.4208	—2.3443	0.7814					0.0000
97–98		0	—1.4830	0.8790	—0.0184	—0.2747							—0.2205
97–n		0	—1.0859	3.2769	—0.7480	—1.0164							0.7369
n–o		0	1.0164	—1.0329	—3.4044	0.5956	2.3443	—0.7814					—0.5711
99			1	—0.5928	0.0124	0.1853							0.1487
100			1	—3.0177	0.6888	0.9360							—0.6786
101			1	—1.0163	—3.3497	0.5860	2.3067	—0.7689					—0.5619
p			1	—3.0000	0.4140	5.9148	—2.3551		0.7850				0.0000
99–100			0	2.4249	—0.6765	—0.7507							0.8274
100–101			0	—2.0014	4.0385	0.3499	—2.3067	0.7689					—0.1167
101–p			0	1.9837	—3.7637	—5.3287	4.6618	—0.7689	—0.7850				—0.5619
102				1	—0.2790	—0.3096							0.3412
103				1	—2.0179	—0.1748	1.1525	—0.3842					0.0583
104				1	—1.8974	—2.6867	2.3505	—0.3877	—0.3953				—0.2833
102–103				0	1.7389	—0.1348	—1.1525	0.3842					0.2829
102–104				0	1.6184	2.3771	—2.3505	0.3877	0.3958				0.6246
105					1	—0.0775	—0.6624	0.2209					0.1627
106					1	1.4690	—1.4525	0.2396	0.2446				0.3859
q					1	1.0210	—4.0419	1.0000	1.0210				—0.6258
r					1		—3.0000	5.0771	0.5385	—3.0000	1.0000		0.0000
105–106					0	—1.5465	0.7901	—0.0186	—0.2446				—0.2232
105–q					0	—1.0985	3.3795	—0.7790	—1.0210				0.7886
q–r					0	1.0210	—1.0419	—4.0771	0.4825	3.0000	—1.0000		—0.6258
107						1	—0.5109	0.0120	0.1581				0.1444
108						1	—3.0762	0.7092	0.9294				—0.7178
109						1	—1.0206	—3.9933	0.4726	2.9383	—0.9795		—0.6130
s						1	—3.0000	0.5274	6.9832	—3.0000		1.0000	0.0000
107–108						0	2.5653	—0.6972	—0.7713				0.8622
108–109						0	—2.0556	4.7025	0.4568	—2.9383	0.9795		—0.1048
109–s						0	1.9794	—4.5207	—6.5106	5.9383	—0.9795	—1.0000	—0.6130
110							1	—0.2718	—0.3006				0.3361
111							1	—2.2875	—0.2222	1.4294	—0.4765		0.0510
112							1	—2.2836	—3.2888	3.0000	—0.4946	—0.5052	—0.3097
110–111							0	2.0157	—0.0784	—1.4294	0.4765		0.2851
111–112							0	2.0118	2.9882	—3.0000	0.4946	5052	0.6457

TABLE 14.—(Continued).

ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 15			Story No. 16			Story No. 17			Story No. 18			
	R_{15}	θ_{A15}	θ_{B15}	R_{16}	θ_{A16}	θ_{B16}	R_{17}	θ_{A17}	θ_{B17}	R_{18}	θ_{A18}	θ_{B18}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												
113		1	−0.0389	−0.7091	0.2364								0.1414
114		1	1.4854	−1.4913	0.2459	0.2511							0.3210
t		1	1.0210	−4.0419	1.0000	1.0210							−0.5209
u		1		−3.0000	4.9092	0.5385	−2.7480	0.9161					0.0000
113–114		0	−1.5243	0.7822	−0.0095	−0.2511							−0.1796
113–t		0	−1.0599	3.3328	−0.7636	−1.0210							0.6624
t–u		0	1.0210	−1.0419	−3.9092	0.4825	2.7480	−0.9161					−0.5209
115			1	−0.5132	0.0062	0.1647							0.1178
116			1	−3.1444	0.7204	0.9633							−0.6249
117			1	−1.0206	−3.8290	0.4726	2.6916	−0.8973					−0.5102
v			1	−3.0000	0.5281	6.8055	−2.7161		0.9054				0.0000
115–116			0	2.6312	−0.7142	−0.7986							0.7427
116–117			0	−2.1238	4.5494	0.4907	−2.6916	0.8973					−0.1147
117–v			0	1.9794	−4.3571	−6.3329	5.4077	−0.8973	−0.9054				−0.5102
118				1	−0.2714	−0.3035							0.2823
119				1	−2.1422	−0.2310	1.2673	−0.4225					0.0540
120				1	−2.2013	−3.1993	2.7318	−0.4533	−0.4574				−0.2579
118–119				0	1.8708	−0.0724	−1.2673	0.4225					0.2282
118–120				0	1.9299	2.8958	−2.7318	0.4533	0.4574				0.5399
121					1	−0.0387	−0.6774	0.2259					0.1220
122					1	1.5027	−1.4177	0.2353	0.2374				0.2802
w					1	1.0092	−4.0214	1.0000	1.0092				−0.4542
x					1		−3.0000	5.1753	0.5878	−3.0000	1.0000		0.0000
121–122					0	−1.5414	0.7403	−0.0094	−0.2374				−0.1582
121–w					0	−1.0479	3.3440	−0.7741	−1.0092				0.5762
w–x					0	1.0092	−1.0214	−4.1755	0.4214	3.0000	−1.0000		−0.4542
123						1	−0.4810	0.0061	0.1542				0.1028
124						1	−3.1958	0.7398	0.9630				−0.5507
125						1	−1.0136	−4.1437	0.4182	2.9773	−0.9924		−0.4507
y						1	−3.0000	0.5833	7.3021	−3.0000		1.0000	0.0000
123–124						0	2.7148	−0.7337	−0.8088				0.6535
124–125						0	−2.1822	4.8835	0.5448	−2.9773	0.9924		−0.0999
125–y						0	1.9864	−4.7270	−6.8839	5.9773	−0.9924	−1.0000	−0.4507
126							1	−0.2703	−0.2979				0.2407
127							1	−2.2380	−0.2496	1.3644	−0.4548		0.0458
128							1	−2.3797	−3.4655	3.0093	−0.4996	−0.5035	−0.2269
126–127							0	1.9677	−0.0483	−1.3644	0.4548		0.1949
126–128							0	2.1094	3.1676	−3.0093	0.4996	0.5035	0.4677

TABLE 14.—(Continued).

ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 17			Story No. 18			Story No. 19			Story No. 20			
	R_{17}	θ_{A17}	θ_{B17}	R_{18}	θ_{A18}	θ_{B18}	R_{19}	θ_{A19}	θ_{B19}	R_{20}	θ_{A20}	θ_{B20}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												
129		1	−0.0245	−0.6935	0.2311								0.0991
130		1	1.5017	−1.4268	0.2369	0.2387							0.2217
z		1	1.0092	−4.0214	1.0000	1.0092							−0.3412
A′		1		−3.0000	5.1755	0.5878	−3.0000	1.0000					0.0000
129–130		0	−1.5262	0.7333	−0.0057	−0.2387							−0.1226
129–z		0	−1.0337	3.3279	−0.7689	−1.0092							0.4403
z–A′		0	1.0092	−1.0214	−4.1755	0.4214	3.0000	−1.0000					−0.3412
131			1	−0.4805	0.0037	0.1564							0.0804
132			1	−3.2197	0.7438	0.9764							−0.4260
133			1	−1.0120	−4.1373	0.4176	2.9727	−0.9909					−0.3381
B′			1	−3.0000	0.5833	7.3021	−3.0000		1.0000				0.0000
131–132			0	2.7392	−0.7401	−0.8200							0.5063
132–133			0	−2.2077	4.8811	0.5588	−2.9727	0.9909					−0.0879
133–B′			0	1.9880	−4.7206	−6.8846	5.9727	−0.9909	−1.0000				−0.3381
134				1	−0.2702	−0.2993							0.1849
135				1	−2.2110	−0.2531	1.3465	−0.4485					0.0398
136				1	−2.3745	−3.4629	3.0042	−0.4980	−0.5030				−0.1701
134–135				0	1.9408	−0.0462	−1.3465	0.4485					0.1451
134–136				0	2.1043	3.1635	−3.0042	0.4980	0.5030				0.3549
137					1	−0.0238	−0.6938	0.2311					0.0747
138					1	1.5034	−1.4277	0.2367	0.2390				0.1687
C′					1	1.0092	−4.0214	1.0000	1.0092				−0.2275
D′					1		−3.0000	5.1755	0.5878	−3.0000	1.0000		0.0000
137–138					0	−1.5272	0.7339	−0.0056	−0.2390				−0.0939
137–C′					0	−1.0330	3.3276	−0.7689	−1.0092				0.3022
C′–D′					0	1.0092	−1.0214	−4.1755	0.4214	3.0000	−1.0000		−0.2275
139						1	−0.4806	0.0036	0.1565				0.0615
140						1	−3.2214	0.7444	0.9769				−0.2926
141						1	−1.0121	−4.1377	0.4176	2.9727	−0.9909		−0.2254
E′						1	−3.0000	0.5833	7.3021	−3.0000		1.0000	0.0000
139–140						0	2.7408	−0.7407	−0.8204				0.3541
140–141						0	−2.2093	4.8821	0.5594	−2.9727	0.9909		−0.0672
141–E′						0	1.9879	−4.7210	−6.8846	5.9727	−0.9909	−1.0000	−0.2254
142							1	−0 2703	−0.2993				0.1292
143							1	−2.2099	−0.2532	1.3455	−0.4485		0.0304
144							1	−2.3749	−3.4632	3.0046	−0.4985	−0.5031	−0.1134
142–143							0	1.9396	−0.0461	−1.3455	0.4485		0.0988
142–144							0	2.1047	3.1639	−3.0046	0.4985	0.5031	0.2426
145								1	−0.0238	−0.6938	0.2313		0.0509
146								1	1.5034	−1.4278	0.2369	0.2390	0.1153
F′								1	1.0092	−4.0214	1.0000	1.0092	−0.1137
G′								1		−3.0000	3.1759	0.5878	0.0000
145–146								0	−1.5272	0.7340	−0.0056	−0.2390	−0.0643
145–F′								0	−1.0330	3.3276	−0.7687	−1.0092	0.1647
F′–G′								0	1.0092	−1.0214	−2.1759	0.4214	−0.1137
147									1	−0.4806	0.0037	0.1565	0.0421
148									1	−3.2217	0.7442	0.9770	−0.1594
149									1	−1.0122	−2.1563	0.4176	−0.1127
H′									1	−3.0000	0.5834	5.3033	0.0000

TABLE 14.—(Continued).

ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation						Right-Hand Member of Equation
	Story No. 19			Story No. 20			
	R_{19}	θ_{A19}	θ_{B19}	R_{20}	θ_{A20}	θ_{B20}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height						Co-efficient of .0001
147–148			0	2.7411	−0.7406	−0.8205	0.2015
148–149			0	−2.2095	2.9005	0.5595	−0.0467
149–H′			0	1.9878	−2.7397	−4.8857	−0.1127
150				1	−0.2702	−0.2993	0.0735
151				1	−1.3129	−0.2531	0.0211
152				1	−1.3783	−2.4580	−0.0567
150–151				0	1.0427	−0.0462	0.0524
150–152				0	1.1081	2.1587	0.1302
153					1	−0.0443	0.0502
154					1	1.9480	0.1175
153–154					•	−1.9923	−0.0673
155						1.0000	0.0338

Table 15.

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

The equations are taken from Table 14.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.				Right-Hand Member of Equation Coefficient of .0001	
155			θ_{B20}	=		.0338
153	$\theta_{A20} - .0443\,\theta_{B20}$			=	.0502	
		.0443 x .00000338		=	.0015	
			θ_{A20}	=		.0517
150	$R_{20} - .2702\,\theta_{A20} - .2993\,\theta_{B20}$			=	.0735	
		.2702 x .00000517		=	.0140	
		.2993 x .00000338		=	.0101	
			R_{20}	=		.0976
147	$\theta_{B19} - .4806\,R_{20} + .0037\,\theta_{A20} + .1565\,\theta_{B20}$			=	.0421	
		.4806 x .00000976		=	0469	.0890
		.0037 x .00000517		=	.0002	
		.1565 x .00000338		=	.0053	—.0055
			θ_{B19}	=		.0835
145	$\theta_{A19} - .0238\,\theta_{B19} - .6938\,R_{20} + .2313\,\theta_{A20}$			=	.0509	
		.0238 x .00000835		=	.0020	
		.6938 x .00000976		=	.0676	.1205
		.2313 x .00000517		=		—.0120
			θ_{A19}	=		.1085
142	$R_{19} - .2703\,\theta_{A19} - .2993\,\theta_{B19}$			=	.1292	
		.2703 x .00001085		=	.0294	
		.2993 x .00000835		=	.0250	
			R_{19}	=		.1836
139	$\theta_{B18} - .4806\,R_{19} + .0037\,\theta_{A19} + .1566\,\theta_{B19}$			=	.0615	
		.4806 x .00001836		=	.0881	.1496
		.0037 x .00001085		=	.0004	
		.1565 x .00000835		=	.0131	—.0135
			θ_{B18}	=		.1361
137	$\theta_{A18} - .0238\,\theta_{B18} - .6938\,R_{19} + .2311\,\theta_{A19}$			=	.0748	
		.0238 x .00001361		=	.0032	
		.6938 x .00001836		=	.1272	.2052
		.2311 x .00001085		=		—.0251
			θ_{A18}	=		.1801
134	$R_{18} - 2702\,\theta_{A18} - .2994\,\theta_{B18}$			=	.1849	
		.2702 x .00001801		=	.0486	
		.2994 x .00001361		=	0407	
			R_{18}	=		.2742

TABLE 15.—(Continued).

DETERMINATION OF THE CHANGES IN THE SLOPES AND OF THE RATIOS OF DEFLECTION TO STORY HEIGHT IN THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
131	$\theta_{B17} - .4805 R_{18} + .0038\ \theta_{A18} + .1564\ \theta_{B18}$		=	.0804	
		.4805 x .00002742	=	.1319	.2123
		.0038 x .00001801	=	.0007	
		.1564 x .00001361	=	.0213	—.0220
		θ_{B17}	=		.1903
129	$\theta_{A17} - .0246\ \theta_{B17} - .6935 R_{18} + .2311\ \theta_{A18}$		=	.0991	
		.0246 x .00001903	=	.0047	
		.6935 x .00002742	=	.1905	2943
		.2311 x .00001801	=		—.0416
		θ_{A17}	=		.2527
126	$R_{17} - .2703\ \theta_{A17} - .2979\ \theta_{B17}$		=	.2407	
		.2703 x .00002527	=	.0682	
		.2979 x .00001903	=	.0566	
		R_{17}	=		.3655
123	$\theta_{B16} - .4810 R_{17} + .0061\ \theta_{A17} + .1542\ \theta_{B17}$		=	.1028	
		.4810 x .00003655	=	.1758	.2786
		.0061 x .00002527	=	.0015	
		.1542 x .00001903	=	.0294	—.0309
		θ_{B16}	=		.2477
121	$\theta_{A16} - .0387\ \theta_{B16} - .6774 R_{17} + .2259\ \theta_{A17}$		=	.1220	
		.0387 x .00002477	=	.0096	
		.6774 x .00003655	=	.2473	.3789
		.2259 x .00002527	=		—.0570
		θ_{A16}	=		.3219
118	$R_{16} - .2714\ \theta_{A16} - .3035\ \theta_{B16}$		=	.2823	
		.2714 x .00003219	=	.0873	
		.3035 x .00002477	=	.0752	
		R_{16}	=		.4448
115	$\theta_{B15} - .5132 R_{16} + .0062\ \theta_{A16} + .1647\ \theta_{B16}$		=	.1178	
		.5132 x .00004448	=	.2280	.3458
		.0062 x .00003219	=	.0020	
		.1647 x .00002477	=	.0408	—.0428
		θ_{B15}	=		.3030
113	$\theta_{A15} - .0389\ \theta_{B15} - .7091 R_{16} + .2364\ \theta_{A16}$		=	.1414	
		.0389 x .00003030	=	.0118	
		.7091 x .00004448	=	.3180	.4712
		.2364 x .00003219	=		—.0762
		θ_{A15}	=		.3950
110	$R_{15} - .2718\ \theta_{A15} - .3006\ \theta_{B15}$		=	.3361	
		.2718 x .00003950	=	.1072	
		.3006 x .00003030	=	.0912	
		R_{15}	=		.5345
107	$\theta_{B14} - .5109 R_{15} + .0120\ \theta_{A15} + .1581\ \theta_{B15}$		=	.1444	
		.5109 x .00005345	=	.2730	.4174
		.0120 x .00003950	=	.0047	
		.1581 x .00003030	=	.0480	—.0527
		θ_{B14}	=		.3647

Table 15.—(Continued).

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
105	$\theta_{A14} - .0775\ \theta_{B14} - .6624\ R_{15} + .2210\ \theta_{A15}$		=	.1627	
		.0775 x .00003647	=	.0283	
		.6624 x .00005345	=	.3541	.5451
		.2210 x .00003950	=		−.0873
		θ_{A14}	=		.4578
102	$R_{14} - .2790\ \theta_{A14} - .3096\ \theta_{B14}$		=	.3412	
		.2790 x .00004578	=	.1278	
		.3096 x .00003647	=	.1160	
		R_{14}	=		.5850
99	$\theta_{B13} - .5928\ R_{14} + .0124\ \theta_{A14} + .1853\ \theta_{B14}$		=	.1487	
		.5928 x .00005850	=	.3470	.4957
		.0124 x .00004578	=	.0057	
		.1853 x .00003647	=	.0676	−.0733
		θ_{B13}	=		.4224
97	$\theta_{A13} - .0695\ \theta_{B13} - .7560\ R_{14} + .2520\ \theta_{A14}$		=	.1658	
		.0695 x .00004224	=	.0294	
		.7560 x .00005850	=	.4420	.6372
		.2520 x .00004578	=		−.1154
		θ_{A13}	=		.5218
94	$R_{13} - .2789\ \theta_{A13} - .3092\ \theta_{B13}$		=	.3864	
		.2789 x .00005218	=	.1458	
		.3092 x .00004224	=	.1308	
		R_{13}	=		.6630
91	$\theta_{B12} - .5920\ R_{13} + .0138\ \theta_{A13} + .1835\ \theta_{B13}$		=	.1715	
		.5920 x .00006630	=	.3921	.5636
		.0138 x .00005218	=	.0072	
		.1835 x .00004224	=	.0775	−.0847
		θ_{B12}	=		.4789
89	$\theta_{A12} - .0766\ \theta_{B12} - .7472\ R_{13} + .2490\ \theta_{A13}$		=	.1816	
		.0766 x .00004789	=	.0366	
		.7472 x .00006630	=	.4960	.7142
		.2490 x .00005218	=		−.1300
		θ_{A12}	=		.5842
86	$R_{12} - .2750\ \theta_{A12} - .3160\ \theta_{B12}$		=	.4216	
		.2750 x .00005842	=	.1603	
		.3160 x .00004789	=	.1511	
		R_{12}	=		.7330
83	$\theta_{B11} - .6150\ R_{12} + .0139\ \theta_{A12} + .1912\ \theta_{B12}$		=	.1831	
		.6150 x .00007330	=	.4512	.6343
		.0139 x .00005842	=	.0081	
		.1912 x .00004789	=	.0916	−.0997
		θ_{B11}	=		.5346
81	$\theta_{A11} - .0770\ \theta_{B11} - .7560\ R_{12} + .2520\ \theta_{A12}$		=	.2018	
		.0770 x .00005346	=	.0411	
		.7560 x .00007330	=	.5550	.7979
		.2520 x .00005842	=		−.1471
		θ_{A11}	=		.6508
78	$R_{11} - .2746\ \theta_{A11} - .3147\ \theta_{B11}$		=	.4669	
		.2746 x .00006508	=	.1790	
		.3147 x .00005346	=	.1681	
		R_{11}	=		.8140

TABLE 15.—(Continued).

DETERMINATION OF THE CHANGES IN THE SLOPES AND OF THE RATIOS OF DEFLECTION TO STORY HEIGHT IN THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
75	$\theta_{B10} - .6061\, R_{11} + .0179\, \theta_{A11} + .1841\, \theta_{B11}$		=	.2178	
		.6061 x .00008140	=	.4940	.7118
		.0179 x .00006508	=	.0116	
		.1841 x .00005346	=	.0984	−.1100
		θ_{B10}	=		.6018
73	$\theta_{A10} - .1082\, \theta_{B10} - .7278\, R_{11} + .2426\, \theta_{A11}$		=	.2055	
		.1082 x .00006018	=	.6651	
		.7278 x .00008140	=	.5925	.8631
		.2426 x .00006508	=		−.1580
		θ_{A10}	=		.7051
70	$R_{10} - .2601\, \theta_{A10} - .3423\, \theta_{B10}$		=	.4540	
		.2601 x .00007051	=	.1833	
		.3423 x .00006018	=	.2059	
		R_{10}	=		.8432
67	$\theta_{B9} - .6991\, R_{10} + .0181\, \theta_{A10} + .2151\, \theta_{B10}$		=	.2056	
		.6991 x .00008432	=	.5890	.7946
		.0181 x .00007051	=	.0127	
		.2151 x .00006018	=	.1293	−.1420
		θ_{B9}	=		.6526
65	$\theta_{A9} - .1038\, \theta_{B9} - .7655\, R_{10} + .2551\, \theta_{A10}$		=	.2053	
		.1038 x .00006526	=	.0677	
		.7655 x .00008432	=	.6460	.9190
		.2551 x .00007051	=		−.1800
		θ_{A9}	=		.7390
62	$R_9 - .2602\, \theta_{A9} - .3372\, \theta_{B9}$		=	.4719	
		.2602 x .00007390	=	.1922	
		.3372 x .00006526	=	.2200	
		R_9	=		.8841
59	$\theta_{B8} - .6908\, R_9 + .0231\, \theta_{A9} + .2072\, \theta_{B9}$		=	.2008	
		.6908 x .00008841	=	.6100	.8108
		.0231 x .00007390	=	.0171	
		.2072 x .00006526	=	.1351	−.1522
		θ_{B8}	=		.6586
57	$\theta_{A8} - .1382\, \theta_{B8} - .7000\, R_9 + .2334\, \theta_{A9}$		=	.2079	
		.1382 x .00006586	=	.0910	
		.7000 x .00008841	=	.6182	.9171
		.2334 x .00007390	=		−.1725
		θ_{A8}	=		.7446
54	$R_8 - .2738\, \theta_{A8} - .3118\, \theta_{B8}$		=	.3774	
		.2738 x .00007446	=	.2038	
		.3118 x .00006586	=	.2055	
		R_8	=		.7867
51	$\theta_{B7} - .5814\, R_8 + .0112\, \theta_{A8} + .1826\, \theta_{B8}$		=	.1562	
		.5814 x .00007867	=	.4570	.6132
		.0112 x .00007446	=	.0083	
		.1826 x .00006586	=	.1203	−.1286
		θ_{B7}	=		.4846

Table 15.—(Continued).

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
49	$\theta_{A7} - .0640\,\theta_{B7} - .7422\,R_8 + .2474\,\theta_{A8}$		=	.1729	
		.0640 x .00004846	=	.0310	
		.7422 x .00007867	=	.5845	.7884
		.2474 x .00007446	=		—.1840
		θ_{A7}	=		.6044
46	$R_7 - .2711\,\theta_{A7} - .3090\,\theta_{B7}$		=	.4011	
		.2711 x .00006044	=	.1639	
		.3090 x .00004846	=	.1497	
		R_7	=		.7147
43	$\theta_{B6} - .5489\,R_7 + .0090\,\theta_{A7} + .1739\,\theta_{B7}$		=	.1706	
		.5489 x .00007147	=	.3920	.5626
		.0090 x .00006044	=	.0054	
		.1739 x .00004846	=	.0842	—.0896
		θ_{B6}	=		.4730
41	$\theta_{A6} - .0540\,\theta_{B6} - .7192\,R_7 + .2397\,\theta_{A7}$		=	.1946	
		.0540 x .00004730	=	.0256	
		.7192 x .00007147	=	.5140	.7342
		.2397 x .00006044	=		—.1448
		θ_{A6}	=		.5894
38	$R_6 - .2648\,\theta_{A6} - .3000\,\theta_{B6}$		=	.4216	
		.2648 x .00005894	=	.1561	
		.3000 x .00004730	=	.1420	
		R_6	=		.7197
35	$\theta_{B5} - .4496\,R_6 + .0013\,\theta_{A6} + .1487\,\theta_{B6}$		=	.1486	
		.4496 x .00007197	=	.3240	.4726
		.0013 x .00005894	=	.0008	
		.1487 x .00004730	=	.0704	—.0712
		θ_{B5}	=		.4014
33	$\theta_{A5} - .0092\,\theta_{B5} - .6676\,R_6 + .2225\,\theta_{A6}$		=	.1879	
		.0092 x .00004014	=	.0037	
		.6676 x .00007197	=	.4802	.6718
		.2225 x .00005894	=		—.1310
		θ_{A5}	=		.5408
30	$R_5 - .2638\,\theta_{A5} - .2981\,\theta_{B5}$		=	.4473	
		.2638 x .00005408	=	.1424	
		.2981 x .00004014	=	.1196	
		R_5	=		.7093
27	$\theta_{B4} - .4348\,R_5 + .0048\,\theta_{A5} + .1401\,\theta_{B5}$		=	.1550	
		.4348 x .00007093	=	.3080	.4630
		.0048 x .00005408	=	.0026	
		.1401 x .00004014	=	.0563	—.0589
		θ_{B4}	=		.4041
25	$\theta_{A4} - .0355\,\theta_{B4} - .6371\,R_5 + .2123\,\theta_{A5}$		=	.2000	
		.0355 x .00004041	=	.0143	
		.6371 x .00007093	=	.4515	.6658
		.2123 x .00005408	=		—.1150
		θ_{A4}	=		.5508

TABLE 15.—(Continued).

DETERMINATION OF THE CHANGES IN THE SLOPES AND OF THE RATIOS OF DEFLECTION TO STORY HEIGHT IN THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation.	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
22	$R_4 + .3642\ \theta_{A4} - .3206\ \theta_{B4} - .4044\ R_5 + .1348\ \theta_{A5}$		=	.5512	
		.3206 x .00004041	=	.1295	
		.4044 x .00007093	=	.2862	.9669
		.3642 x .00005508	=	.2008	
		.1348 x .00005408	=	.0729	—.2737
		R_4	=		.6932
19	$\theta_{B3} - .4696\ R_4 + .0030\ \theta_{A4} + .1535\ \theta_{B4}$		=	.1540	
		.4696 x .00006932	=	.3257	.4797
		.0030 x .00005508	=	.0017	
		.1535 x .00004041	=	.0620	—.0637
		θ_{B3}	=		.4160
17	$\theta_{A3} - .0196\ \theta_{B3} - .6881\ R_4 + .2294\ \theta_{A4}$		=	.1917	
		.0196 x .00004160	=	.0082	
		.6881 x .00006932	=	.4770	.6769
		.2294 x .00005508	=		—.1263
		θ_{A3}	=		.5506
14	$R_3 - .2689\ \theta_{A3} - .2980\ \theta_{B3}$		=	.4564	
		.2689 x .00005506	=	.1481	
		.2980 x .00004160	=	.1240	
		R_3	=		.7285
11	$\theta_{B2} - .4511\ R_3 + .0027\ \theta_{A3} + .1476\ \theta_{B3}$		=	.1768	
		.4511 x .00007285	=	.3285	.5053
		.0027 x .00005506	=	.0015	
		.1476 x .00004160	=	.0614	—.0629
		θ_{B2}	=		.4424
9	$\theta_{A2} - .0191\ \theta_{B2} - .6857\ R_3 + .2286\ \theta_{A3}$		=	.2151	
		.0191 x .00004424	=	.0085	
		.6857 x .00007285	=	.4990	.7226
		.2286 x .00005506	=		—.1259
		θ_{A2}	=		.5967
6	$R_2 - .2627\ \theta_{A2} - .2918\ \theta_{B2}$		=	.5021	
		.2627 x .00005967	=	.1565	
		.2918 x .00004424	=	.1291	
		R_2	=		.7877
3	$\theta_{B1} - .3612\ R_2 - .0088\ \theta_{A2} + .1292\ \theta_{B2}$		=	.1483	
		.3612 x .00007877	=	.2845	
		.0088 x .00005967	=	.0053	.4381
		.1292 x .00004424	=		—.0572
		θ_{B1}	=		.3809
1	$\theta_{A1} + .0680\ \theta_{B1} - .6494\ R_2 + .2164\ \theta_{A2}$		=	.2667	
		.6494 x .00007877	=	.5110	.7777
		.0680 x .00003809	=	.0259	
		.2164 x .00005967	=	.1291	—.1550
		θ_{A1}	=		.6227
A	$R_1 - .2495\ \theta_{A1} - .2495\ \theta_{B1}$		=	.5667	
		.2495 x .00006227	=	.1553	
		.2495 x .00003809	=	.0951	
		R_1	=		.8171

TABLE 16.

VALUES OF R AND θ FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5, AND THE FUNCTIONS OF THESE VALUES THAT OCCUR IN THE EQUATIONS USED TO DETERMINE THE MOMENTS IN THE COLUMNS AND GIRDERS.

Story No.	10,000 x R	10,000 x θ_A	10,000 x θ_B	10,000 x $3R$	10,000 x $2\theta_A$	10,000 x $2\theta_B$	10,000 x $(2\theta_A+\theta_B)$	10,000 x $(2\theta_B+\theta_A)$	10,000 x $3\theta_B$	For Column A*: 10,000 x $(2\theta_{N-1}+\theta_N-3R)$	For Column A*: 10,000 x $(2\theta_N+\theta_{N-1}-3R)$	For Column B: 10,000 x $(2\theta_{N-1}+\theta_N-3R)$	For Column B: 10,000 x $(2\theta_N+\theta_{N-1}-3R)$	Story No.
1	.8171	.6227	.3809	2.4413	1.2454	.7618	1.6263	1.3845	1.1427	1.8186	1.1959	2.0604	1.6795	1
2	.7877	.5967	.4424	2.3631	1.1934	.8848	1.6358	1.4815	1.3372	.5210	.5470	1.1589	1.0974	2
3	.7285	.5506	.4160	2.1855	1.1012	8320	1.5172	1.3826	1.2480	.4415	.4876	.8847	.9111	3
4	.6932	.5508	.4041	2.0796	1.1016	.8082	1.5057	1.3590	1.2123	.4276	.4274	.8435	.8554	4
5	.7093	.5408	.4014	2.1279	1.0816	.8028	1.4830	1.3436	1.2042	.4855	.4955	.9183	.9210	5
6	.7197	.5894	.4730	2.1591	1.1788	.9460	1.6518	1.5354	1.4190	.4881	.4395	.8833	.8117	6
7	.7147	.6044	.4846	2.1441	1.2088	.9692	1.6934	1.5736	1.4538	.3609	.3459	.7135	.7019	7
8	.7867	.7446	.6586	2.3601	1.4892	1.3172	2.1478	2.0618	1.9758	.4067	.2665	.7323	.5583	8
9	.8841	.7390	.6526	2.6523	1.4780	1.3052	2.1306	2.0442	1.9578	.4241	.4297	.6825	.6885	9
10	.8432	.7051	.6018	2.5296	1 4102	1.2036	2.0120	1.9087	1.8054	.3465	.3804	.6226	.6734	10
11	.8140	.6508	.5346	2.4420	1.3016	1.0692	1 8362	1 7200	1.6038	.3810	.4353	.7038	.7710	11
12	.7330	.5842	.4789	2.1990	1.1684	.9578	1.6473	1.5420	1.4367	.3132	.3798	.6509	.7066	12
13	.6630	.5218	.4224	1.9890	1.0436	.8448	1.4660	1.3666	1.2682	.2988	.3612	.6088	.6653	13
14	.5850	.4578	.3647	1.7550	.9156	.7294	1.2803	1.1872	1.0941	.2536	.3176	.5455	.6032	14
15	.5345	.3950	.3030	1.6035	.7900	.6060	1.0930	1.0010	.9090	.2929	.3557	.5711	.6328	15
16	.4448	.3219	.2477	1.3344	.6438	.4954	.8915	.8173	.7431	.2225	.2956	.4807	.5360	16
17	.3655	.2527	.1903	1.0965	.5054	.3806	.6957	.6333	.5709	.2000	.2692	.4108	.4682	17
18	.2742	.1801	.1361	.8226	.3602	.2722	.4963	.4523	.4083	.1371	.2097	.3059	.3601	18
19	.1836	.1085	.0335	.5508	.2170	.1670	.3005	.2755	.2505	.0821	.1537	.1951	.2477	19
20	.0976	.0517	.0338	.2928	.1034	.0676	.1372	.1193	.1014	.0231	.0809	.0920	.1417	20

* N represents the number of the story in question, and (N—1) the story below.

TABLE 17.

VALUES OF K FOR THE COLUMNS AND THE GIRDERS OF THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5, AND THE FUNCTIONS OF THESE VALUES THAT OCCUR IN THE EQUATIONS USED TO DETERMINE THE MOMENTS IN THE COLUMNS AND GIRDERS.

Story No.	K for Girder at Top of Story in		K for		.0001 x $2EK$ for Girder at Top of Story in		.0001 x $2EK$ for	
	Bay a	Bay b	Column A	Column B	Bay a	Bay b	Column A	Column B
1	30.5	37.3	25.8	25.8	177 000	216 200	149 600	149 600
2	21.4	29.2	35.6	35.6	124 100	169 400	206 300	206 300
3	21.4	26.2	35.4	35.5	124 100	152 000	205 500	205 800
4	19.5	26.2	35.4	35.5	113 050	152 000	205 500	205 800
5	19.5	23.8	29.4	30.4	113 050	138 050	170 500	176 300
6	14.1	17.2	29.4	30.4	81 750	99 700	170 500	176 300
7	12.8	15.7	28.7	30.0	74 250	91 100	166 500	174 000
8	7.7	9.4	28.7	30.0	44 600	54 500	166 500	174 000
9	7.7	9.4	21.1	26.1	44 600	54 500	122 400	151 300
10	7.7	9.4	21.1	26.1	44 600	54 500	122 400	151 300
11	7.7	9.4	18.8	19.9	44 600	54 500	109 000	115 400
12	7.7	9.4	18.8	19.9	44 600	54 500	109 000	115 400
13	7.7	9.4	18.3	18.6	44 600	54 500	106 100	107 900
14	7.7	9.4	18.3	18.6	44 600	54 500	106 100	107 900
15	7.7	9.4	14.3	14.6	44 600	54 500	82 900	84 600
16	7.7	9.4	14 3	14.6	44 600	54 500	82 900	84 600
17	7.7	9.4	13.1	13.2	44 600	54 500	76 000	76 500
18	7.7	9.4	13.1	13.2	44 600	54 500	76 000	76 500
19	7.7	9.4	13.1	13.2	44 600	54 500	76 000	76 500
20	7.7	9.4	13.1	13.2	44 600	54 500	76 000	76 500

Table 18.

Moments at the Ends of the Columns and Girders of the Symmetrical Three-span Twenty-story Bent shown in Fig. 5.

Moments are expressed in inch-pounds.

No. of Story	Moment in Column A*		Moment in Column B		Moment in Girder at Top of Story in Bay a		Moment in Girder at Top of Story in Bay b
	At Top = 2EK $(2\theta_N+\theta_{N-1}-3R)$	At Bottom = 2EK $(2\theta_{N-1}+\theta_N-3R)$	At Top = 2EK $(2\theta_N+\theta_{N-1}-3R)$	At Bottom = 2EK $(2\theta_{N-1}+\theta_N-3R)$	At Right-hand End = 2EK $(2\theta_A+\theta_B)$	At Left-hand End = 2EK $(2\theta_B+\theta_A)$	At Each End = 2EK $(3\theta_B)$
1	178 800	272 000	251 000	308 200	287 800	245 000	247 500
2	113 000	107 500	226 500	239 000	203 000	184 000	226 000
3	100 300	90 700	187 500	182 000	187 100	171 700	189 600
4	87 800	87 800	176 100	173 500	170 500	153 600	184 400
5	84 500	82 800	162 300	162 000	167 700	152 000	166 000
6	75 000	83 200	143 300	155 700	135 000	125 600	141 500
7	57 600	60 000	122 000	124 000	125 900	117 000	132 500
8	44 400	67 700	97 200	127 500	95 800	92 000	107 800
9	52 550	51 900	104 200	103 300	95 000	91 100	106 800
10	46 500	42 400	102 000	94 200	89 600	85 000	98 500
11	47 500	41 500	89 000	81 200	81 900	76 600	87 500
12	41 400	34 100	81 500	75 100	73 400	68 600	78 300
13	38 300	31 700	71 800	65 600	65 300	60 800	69 100
14	33 700	26 900	65 100	58 900	5 700	52 800	59 600
15	29 420	24 300	53 600	48 350	48 700	49 600	49 600
16	24 450	18 450	45 400	40 600	39 700	36 400	40 500
17	20 450	15 400	35 800	31 400	31 000	28 200	31 100
18	15 920	10 430	27 600	23 400	22 100	20 150	22 300
19	11 680	6 240	18 900	14 930	13 380	12 250	13 650
20	6 150	1 750	10 830	7 040	6 110	5 320	5 530

*N represents the number of the story in question, and (N—1) the story below.

TABLE 19.

DIRECT STRESSES IN THE COLUMNS, AND THE SHEARS IN THE COLUMNS AND GIRDERS OF THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

All quantities are in pounds.

Story No.	Shear in		Shear in Girder at Top of Story in		Direct Stress in	
	Column *A*	Column *B*	Bay *a*	Bay *b*	Column *A*	Column *B*
1	1 709	2 115	2 020	2 290	14 464	4 587
2	1 149	2 421	1 467	2 095	12 444	4 317
3	1 136	2 200	1 360	1 755	10 977	3 689
4	1 044	2 080	1 228	1 709	9 617	3 294
5	995	1 931	1 210	1 536	8 389	2 813
6	940	1 780	986	1 310	7 179	2 489
7	816	1 709	920	1 228	6 193	2 163
8	779	1 560	712	996	5 273	1 855
9	725	1 440	705	988	4 561	1 571
10	617	1 363	661	912	3 856	1 288
11	618	1 180	600	810	3 195	1 037
12	524	1 088	538	725	2 595	827
13	486	954	478	640	2 057	640
14	421	861	416	552	1 579	478
15	373	710	354	459	1 163	342
16	298	597	288	375	809	237
17	249	466	221	288	521	150
18	182	354	160	206	300	83
19	124	235	97	126	140	37
20	55	124	43	51	43	8

TABLE 20.

CHECK ON THE NUMERICAL VALUES OF THE MOMENTS AT THE ENDS OF THE COLUMNS AND GIRDERS OF THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

Quantities are in inch-pounds.

For a perfect check the corresponding quantities in the two columns of each group of columns should be identical.

Story No.	Total Shear in a Story Multiplied by the Story Height	Sum of the Moments at the Tops and the Bottoms of all of the Columns in a Story	Sum of the Moments in Column *A* just above and below Girder *a* at the Top of a Story	Moment in Girder *a* at the Top of a Story, at the Section adjacent to Column *A*	Sum of the Moments in Column *B* just above and below Girders *a* and *b* at the Top of a Story	Sum of the Moments in Girders *a* and *b* at the Top of a Story at Sections adjacent to Column *B*
1	2 035 000	2 020 000	286 300	287 800	490 000	492 500
2	1 370 000	1 372 000	203 700	203 000	408 500	410 000
3	1 123 000	1 121 000	187 100	188 100	361 000	361 300
4	1 053 000	1 050 400	170 500	170 600	338 100	338 000
5	983 000	983 200	167 700	167 700	318 000	318 000
6	913 000	914 400	135 000	135 000	267 300	267 100
7	725 000	727 200	125 900	125 300	249 500	249 500
8	673 000	673 600	95 800	96 300	200 500	199 800
9	622 000	624 000	95 000	94 950	198 400	197 900
10	570 000	570 200	89 600	88 000	183 200	183 500
11	518 000	518 400	81 900	81 600	164 100	164 100
12	466 000	464 200	73 400	72 100	147 100	146 900
13	414 000	414 800	65 300	65 200	130 700	129 900
14	363 000	369 200	57 000	58 000	113 450	112 400
15	311 000	311 400	48 700	47 870	94 200	94 200
16	259 000	259 800	39 700	39 850	76 800	76 900
17	207 500	206 200	31 000	30 880	59 200	59 300
18	155 300	154 800	22 100	22 160	42 530	42 450
19	103 600	103 500	13 380	13 430	25 940	25 900
20	51 800	51 540	6 110	6 150	10 830	10 850

Table 21.

Elimination of the Unknown Quantities in the Equations Used to Determine the Slopes and the Deflections in the Bottom Story of the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5, by a Modification of the Slope-Deflection Method.

No. of Equation	Left-Hand Member of Equation							Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3	
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height							Co-efficient of .0001
A	−206.8	51.6	51.6					−117.2
B	− 77.4	183.8	30.5	−106.8	35.6			0.0
C	− 77.4	30.5	295.7	−106.8		35.6		0.0
D		71.2	71.2	−284.8	71.2	71.2		− 78.9
E		35.6		−106.8	220.2	21.4	−106.2	0.0
F			35.6	−106.8	21.4	308.1	−106.5	0.0
G					141.6	142.0	−283.6	− 64.5
A	1	−0.2495	−0.2495					0.5667
B	1	−2.3 47	−0.3941	1.3798	−0.4599			0.0000
C	1	−0.3941	−3.8204	1.3798		−0.4599		0.0000
A–B	0	2.1252	0.1445	−1.3798	0.4599			0.5667
B–C	0	−1.9807	3.4264		−0.4599	0.4599		0.0000
1		1	0.0680	−0.6494	0.2164			0.2667
2		1	−1.7300		0.2322	−0.2322		0.0000
D		1	1.0000	−4.0000	1.0000	1.0000		−1.1082
E		1		−3.0000	6.1856	0.6012	−2.9834	0.0000
1–2		0	1.7980	−0.6494	−0.0158	0.2322		0.2667
2–D		0	−2.7300	4.0000	−0.7678	−1.2322		1.1082
D–E		0	1.0000	−1.0000	−5.1856	0.3988	2.9834	−1.1082
3			1	−0.3612	−0.0088	0.1292		0.1483
4			1	−1.4652	0.2812	0.4513		−0.4059
5			1	−1.0000	−5.1856	0.3988	2.9834	−1.1082
F			1	−3.0000	0.6012	8.6547	−2.9915	0.0000
3–4			0	1.1041	−0.2900	−0.3222		0.5543
4–5			0	−0.4652	5.4668	0.0525	−2.9834	0.7023
5–F			0	2.0000	−5.7868	8.2559	5.9749	−1.1082
6				1	−0.2627	−0.2918		0.5021
7				1	−11.7518	−0.1128	6.4127	−1.5095
8				1	−2.8934	−4.1279	2.9875	−0.5541
6–7				0	11.4891	−0.1790	−6.4127	2.0116
7–8				0	−8.8554	4.0151	3.4252	−0.9554
9					1	−0.0156	−0.5600	0.1750
10					1	−0.4540	−0.3862	0.1079
G					1	1.0030	−2.0010	−0.4555
9–10					0	0.4384	−0.1738	0.0671
10–G					0	−1.4570	1.6148	0.5634
11						1	−0.3961	0.1530
12						1	−1.1090	−0.3865
11–12						0	0.7131	0.5395
13							1	0.7560

Table 22.

Determination of the Changes in the Slopes and the Ratio of the Deflection to Story Height in the Bottom Story of the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5, by a Modification of the Slope-Deflection Method.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.				Right-Hand Member of Equation Coefficient of .0001	
13			R_3	=	.7560	
11	$\theta_{B2} - .3961 R_3$			=	.1530	
		.3961 x .00007560		=	.2995	
			θ_{B2}	=		.4525
9	$\theta_{A2} - .0156\ \theta_{B2} - .5600\ R_3$			=	.1750	
		.0156 x .00004525		=	.0071	
		.5600 x .00007560		=	.4240	
			θ_{A2}	=		.6061
6	$R_2 - .2627\ \theta_{A2} - .2918\ \theta_{B2}$			=	.5021	
		.2627 x .00006061		=	.1593	
		.2918 x .00004525		=	.1320	
			R_2	=		.7934
3	$\theta_{B1} - .3612\ R_2 - .0088\ \theta_{A2} + .1292\ \theta_{B2}$			=	.1483	
		.3612 x .00007934		=	.2865	
		.0088 x .00006061		=	.0053	.4401
		.1292 x .00004525		=		—.0585
			θ_{B1}	=		.3816
1	$\theta_{A1} + .0680\ \theta_{B1} - .6494\ R_2 + .2164\ \theta_{A2}$			=	.2667	
		.6494 x .00007934		=	.5150	.7817
		.0680 x .00003816		=	.0260	
		.2164 x .00006061		=	.1313	—.1573
			θ_{A1}	=		.6244
A	$R_1 - .2495\ \theta_{A1} - .2495\ \theta_{B1}$			=	.5667	
		.2495 x .00006244		=	.1556	
		.2495 x .00003816		=	.0951	
			R_1	=		.8174

TABLE 23.

Comparison of the Approximate Methods with the Slope-Deflection Method When Applied to the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

Moments in the Columns.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Story No.	Moment at Top of Column *A*					Moment at Bottom of Column *A*					Moment at Top of Column *B*					Moment at Bottom of Column *B*					Story No.
	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method I	Fleming's Method II	Fleming's Method III	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method I	Fleming's Method II	Fleming's Method III	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method I	Fleming's Method II	Fleming's Method III	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method I	Fleming's Method II	Fleming's Method III	
1	178.8	191.5	166.9	254.5	254.5	272.0	191.5	166.9	254.5	254.5	251.0	319.0	343.2	254.5	254.5	308.2	319.0	343.2	254.2	254.5	1
	100.0	*106.8*	*93.7*	*145.0*	*145.0*	*100.0*	*70.3*	*61.3*	*93.6*	*93.6*	*100.0*	*126.8*	*136.9*	*101.4*	*101.4*	*100.0*	*103.5*	*111.3*	*82.5*	*82.5*	
2	113.0	113.8	111.0	171.4	171.4	107.5	113.8	111.0	171.4	171.4	226.5	227.5	228.6	171.4	171.4	239.0	227.5	228.6	171.4	171.4	2
	100.0	*100.7*	*98.3*	*152.0*	*152.0*	*100.0*	*106.0*	*103.4*	*159.4*	*159.4*	*100.0*	*100.4*	*101.0*	*75.7*	*75.7*	*100.0*	*95.3*	*95.8*	*71.7*	*71.7*	
3	100.3	96.8	91.5	140.5	140.5	90.7	96.8	91.5	140.5	140.5	187.5	184.1	187.6	140.5	140.5	182.0	184.1	187.6	140.5	140.5	3
	100.0	*96.5*	*91.2*	*140.3*	*140.3*	*100.0*	*106.8*	*101.0*	*155.0*	*155.0*	*100.0*	*98.3*	*100.0*	*75.0*	*75.0*	*100.0*	*101.2*	*103.0*	*77.2*	*77.2*	
4	87.8	87.1	87.1	131.7	131.7	87.8	87.1	87.1	131.7	131.7	176.1	175.0	175.3	131.7	131.7	173.5	175.0	175.3	131.7	131.7	4
	100.0	*99.3*	*99.2*	*150.0*	*150.0*	*100.0*	*99.3*	*99.2*	*150.0*	*150.0*	*100.0*	*99.4*	*99.5*	*74.6*	*74.6*	*100.0*	*101.0*	*101.2*	*75.9*	*75.9*	
5	84.5	85.6	80.9	120.4	120.4	82.8	85.6	80.9	120.4	120.4	162.3	160.0	163.1	125.3	125.3	162.0	160.0	163.1	125.3	125.3	5
	100.0	*101.3*	*95.6*	*142.3*	*142.3*	*100.0*	*103.3*	*97.6*	*145.3*	*145.3*	*100.0*	*98.5*	*100.6*	*77.2*	*77.2*	*100.0*	*98.8*	*100.8*	*77.5*	*77.5*	
6	75.0	76.8	74.3	111.7	111.7	83.2	76.8	74.3	111.7	111.7	143.3	151.8	153.0	116.3	116.3	155.7	151.8	153.0	116.3	116.3	6
	100.0	*102.0*	*99.0*	*149.0*	*149.0*	*100.0*	*92.2*	*89.5*	*134.2*	*134.2*	*100.0*	*106.0*	*106.8*	*81.1*	*81.1*	*100.0*	*97.5*	*98.3*	*74.8*	*74.8*	
7	57.6	60.3	59.2	88.6	88.6	60.0	60.3	59.2	88.6	88.6	122.0	120.7	122.0	92.8	92.8	124.0	120.7	122.0	92.8	92.8	7
	100.0	*105.0*	*102.8*	*154.0*	*154.0*	*100.0*	*100.5*	*98.5*	*147.6*	*147.6*	*100.0*	*98.8*	*100.0*	*76.1*	*76.1*	*100.0*	*97.3*	*98.3*	*74.8*	*74.8*	
8	44.4	54.7	55.9	82.3	82.3	67.7	54.7	55.9	82.3	82.3	97.2	114.0	113.0	86.2	86.2	127.5	114.0	113.0	86.2	86.2	8
	100.0	*125.0*	*126.0*	*185.5*	*185.5*	*100.0*	*80.7*	*82.5*	*121.8*	*121.8*	*100.0*	*117.2*	*116.2*	*88.7*	*88.7*	*100.0*	*89.5*	*88.6*	*67.6*	*67.6*	
9	52.6	49.8	51.4	69.5	69.5	51.9	49.8	51.4	69.5	69.5	104.2	105.5	104.5	86.0	86.0	103.3	105.5	104.5	86.0	86.0	9
	100.0	*94.8*	*97.7*	*132.0*	*132.0*	*100.0*	*96.1*	*99.1*	*134.0*	*134.0*	*100.0*	*100.8*	*100.3*	*82.5*	*82.5*	*100.0*	*102.1*	*101.0*	*83.2*	*83.2*	
10	46.5	45.6	46.0	63.7	63.7	42.4	45.6	46.0	63.7	63.7	102.2	96.3	96.8	78.8	78.8	94.2	96.3	96.8	78.8	78.8	10
	100.0	*98.0*	*99.0*	*137.0*	*137.0*	*100.0*	*107.8*	*108.5*	*150.4*	*150.4*	*100.0*	*94.3*	*94.9*	*77.3*	*77.3*	*100.0*	*102.1*	*102.7*	*83.6*	*83.6*	
11	47.5	43.0	41.9	62.9	62.9	41.5	43.0	41.9	62.9	62.9	89.0	86.4	88.1	66.7	66.7	81.2	86.4	88.1	66.7	66.7	11
	100.0	*90.6*	*88.1*	*132.3*	*132.3*	*100.0*	*103.8*	*101.0*	*152.8*	*152.8*	*100.0*	*97.1*	*99.0*	*74.9*	*74.9*	*100.0*	*106.5*	*108.5*	*82.0*	*82.0*	
12	41.4	38.7	38.2	56.7	56.7	34.1	38.7	38.2	56.7	56.7	81.5	77.9	79.1	60.0	60.0	75.1	77.9	79.1	60.0	60.0	12
	100.0	*93.6*	*92.4*	*137.0*	*137.0*	*100.0*	*113.7*	*112.0*	*166.4*	*166.4*	*100.0*	*95.6*	*97.0*	*73.6*	*73.6*	*100.0*	*103.5*	*105.4*	*79.9*	*79.9*	

TABLE 24.

COMPARISON OF THE APPROXIMATE METHODS WITH THE SLOPE-DEFLECTION METHOD WHEN APPLIED TO THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

MOMENTS IN THE GIRDERS.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Story No.	Moment at Right End of Girder *a*					Moment at Left End of Girder *a*					Moment at End of Girder *b*					Story No.
	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			
			I	II	III			I	II	III			I	II	III	
1	287.8 *100.0*	305.3 *106.1*	279.3 *97.1*	425.9 *148.0*	425.9 *148.0*	245.0 *100.0*	274.5 *112.0*	279.3 *114.0*	143.7 *58.6*	130.9 *53.4*	247.5 *100.0*	272.0 *110.0*	295.0 *119.0*	282.2 *114.0*	295.0 *119.0*	1
2	203.0 *100.0*	210.6 *103.5*	199.8 *98.5*	311.9 *153.6*	311.9 *153.6*	184.0 *100.0*	185.6 *100.8*	199.8 *108.5*	104.4 *56.7*	100.7 *54.8*	226.0 *100.0*	226.0 *100.0*	211.2 *93.4*	207.5 *91.9*	211.2 *93.4*	2
3	187.1 *100.0*	183.9 *98.2*	178.2 *95.2*	272.2 *145.8*	272.2 *145.8*	171.7 *100.0*	173.1 *100.8*	178.2 *104.0*	90.7 *52.9*	84.3 *49.1*	189.6 *100.0*	186.0 *98.0*	187.9 *99.0*	181.5 *95.8*	187.9 *99.0*	3
4	170.5 *100.0*	172.7 *101.0*	169.2 *99.5*	252.1 *148.0*	252.1 *148.0*	153.6 *100.0*	152.0 *99.0*	169.2 *110.3*	87.3 *56.9*	84.9 *55.3*	184.4 *100.0*	183.0 *99.0*	172.1 *93.5*	169.7 *92.0*	172.1 *93.5*	4
5	167.7 *100.0*	162.4 *97.0*	154.8 *92.4*	232.1 *138.7*	232.1 *138.7*	152.0 *100.0*	151.8 *99.9*	154.8 *101.8*	83.7 *55.0*	77.3 *50.9*	166.0 *100.0*	160.0 *96.5*	164.4 *99.0*	157.9 *95.0*	164.4 *99.0*	5
6	135.0 *100.0*	137.1 *101.2*	133.2 *98.8*	200.3 *148.2*	200.3 *148.2*	125.6 *100.0*	128.5 *102.6*	133.2 *112.2*	73.0 *58.1*	68.3 *54.4*	141.5 *100.0*	144.0 *102.0*	140.8 *99.5*	136.1 *96.3*	140.8 *99.5*	6
7	125.9 *100.0*	115.0 *91.5*	114.5 *91.0*	170.9 *135.9*	170.9 *135.9*	117.0 *100.0*	110.9 *94.9*	114.5 *98.0*	62.4 *53.3*	57.4 *49.1*	132.5 *100.0*	123.8 *93.0*	121.6 *91.8*	116.6 *88.0*	121.6 *91.8*	7
8	95.8 *100.0*	104.5 *109.3*	108.7 *113.3*	151.8 *158.4*	151.8 *158.4*	92.0 *100.0*	103.5 *112.5*	108.7 *118.0*	64.2 *69.8*	56.0 *60.9*	107.8 *100.0*	116.0 *108.0*	106.2 *95.8*	108.0 *100.3*	106.2 *95.8*	8
9	95.0 *100.0*	95.4 *100.4*	96.8 *101.9*	133.2 *140.3*	133.2 *140.3*	91.1 *100.0*	95.8 *105.0*	96.8 *112.4*	65.5 *71.9*	60.5 *66.4*	106.8 *100.0*	103.0 *99.4*	104.3 *97.6*	99.3 *93.0*	104.3 *97.6*	9
10	89.6 *100.0*	88.6 *98.9*	87.1 *97.3*	126.6 *141.2*	126.6 *141.2*	85.0 *100.0*	85.7 *100.8*	87.1 *102.4*	54.8 *64.5*	48.0 *56.5*	98.5 *100.0*	97.0 *98.5*	97.5 *99.0*	90.7 *92.1*	97.5 *99.0*	10
11	81.9 *100.0*	81.7 *99.8*	80.3 *98.2*	119.6 *146.2*	119.6 *146.2*	76.6 *100.0*	77.0 *100.4*	80.3 *104.8*	44.6 *58.2*	40.8 *53.3*	87.5 *100.0*	87.3 *99.8*	85.9 *98.1*	82.1 *94.0*	85.9 *98.1*	11
12	73.4 *100.0*	73.1 *99.6*	72.3 *98.7*	108.0 *147.2*	108.0 *147.2*	68.6 *100.0*	68.5 *100.0*	72.3 *105.5*	38.9 *56.8*	36.4 *53.0*	78.3 *100.0*	78.1 *99.8*	75.9 *96.9*	73.4 *93.8*	75.9 *96.9*	12

TABLE 25.

COMPARISON OF THE APPROXIMATE METHODS WITH THE SLOPE-DEFLECTION METHOD WHEN APPLIED TO THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 13.

MOMENTS IN THE COLUMNS.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Story No.	Moment at Top of Column *A*					Moment at Bottom of Column *A*					Moment at Top of Column *B*					Moment at Bottom of Column *B*					Story No.
	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			
			I	II	III			I	II	III			I	II	III			I	II	III	
1	193.2	203.5	138.0	254.4	254.4	275.9	203.5	138.0	254.4	254.4	248.0	305.0	370.5	254.4	254.4	302.8	305.0	370.5	254.4	254.4	1
	100.0	*105.5*	*71.4*	*131.5*	*131.5*	*100.0*	*73.8*	*50.0*	*92.0*	*92.0*	*100.0*	*122.8*	*149.5*	*102.2*	*102.2*	*100.0*	*100.5*	*122.0*	*84.0*	*80.4*	
2	125.9	127.5	92.5	171.4	171.4	123.8	127.5	92.5	171.4	171.4	212.7	214.0	248.5	171.4	171.4	223.7	214.0	248.5	171.4	171.4	2
	100.0	*101.3*	*73.5*	*136.0*	*136.0*	*100.0*	*102.8*	*75.0*	*148.2*	*148.2*	*100.0*	*100.5*	*117.0*	*80.5*	*80.5*	*100.0*	*95.8*	*111.3*	*76.8*	*76.8*	
3	110.6	108.0	75.9	140.5	140.5	101.6	108.0	75.9	140.5	140.5	177.1	174.0	203.9	140.5	140.5	171.6	174.0	203.9	140.5	140.5	3
	100.0	*97.5*	*68.7*	*127.0*	*127.0*	*100.0*	*106.5*	*74.7*	*138.2*	*138.2*	*100.0*	*98.2*	*114.9*	*79.3*	*79.3*	*100.0*	*101.5*	*118.6*	*82.0*	*82.0*	
4	99.4	97.8	71.5	131.7	131.7	99.4	97.8	71.5	131.7	131.7	165.3	164.1	191.9	131.7	131.7	163.7	164.1	191.9	131.7	131.7	4
	100.0	*98.3*	*72.0*	*132.2*	*132.2*	*100.0*	*98.3*	*72.0*	*132.0*	*132.0*	*100.0*	*99.5*	*116.0*	*79.6*	*79.6*	*100.0*	*100.3*	*117.5*	*80.5*	*80.5*	

Table 26

Comparison of the Approximate Methods with the Slope-Deflection Method When Applied to the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 13.

Moments in the Girders.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Story No.	Moment at Right End of Girder *a*					Moment at Left End of Girder *a*					Moment at End of Girder *b*					Story No.
	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			
			I	II	III			I	II	III			I	II	III	
1	316.9 *100.0*	331.0 *104.5*	230.9 *73.0*	425.8 *135.2*	425.8 *135.2*	277.4 *100.0*	309.0 *111.5*	230.9 *83.3*	138.5 *50.0*	36.0 *13.0*	194.5 *100.0*	210.0 *108.0*	389.2 *200.0*	287.3 *148.0*	389.8 *200.0*	1
2	227.6 *100.0*	235.5 *103.8*	167.7 *73.5*	311.9 *137.0*	311.9 *137.0*	209.4 *100.0*	212.2 *101.9*	167.7 *80.0*	103.0 *49.4*	23.6 *11.3*	174.6 *100.0*	175.8 *100.5*	282.8 *162.0*	208.8 *119.6*	288.2 *165.0*	2
3	210.1 *100.0*	205.8 *98.0*	147.7 *70.0*	272.2 *129.5*	272.2 *129.5*	194.7 *100.0*	194.1 *99.7*	147.7 *75.8*	90.7 *46.6*	23.2 *11.9*	146.4 *100.0*	144.0 *98.2*	249.1 *170.0*	181.4 *124.0*	249.0 *170.0*	3
4	191.0 *100.0*	192.3 *100.8*	138.1 *72.3*	254.5 *133.0*	254.5 *133.0*	175.6 *100.0*	173.3 *98.8*	138.1 *79.0*	84.8 *48.3*	21.7 *12.4*	143.3 *100.0*	143.0 *98.8*	232.9 *162.2*	169.7 *118.1*	232.8 *162.2*	4

TABLE 27.

EFFECT OF THE PROPORTIONS OF A BENT UPON THE ACCURACY OF METHOD I.

ALL GIRDER SECTIONS ARE THE SAME.

All stories of a bent are identical, and the shears on all stories are equal. All column sections are equal, all girder sections are equal, and the column sections are equal to the girder sections.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column A		Moment at Top and Bottom of Column B		Moment at Right End of Girder a		Moment at Left End of Girder a		Moment at End of Girder b	
	Ratio of Story Height to Width of Bay a	Ratio of Width of Bay a to Width of Bay b	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I
1	2	2	9.09 *100.0*	9.62 *105.9*	15.91 *100.0*	15.38 *96.5*	18.20 *100.0*	19.24 *105.9*	13.67 *100.0*	19.24 *140.7*	18.20 *100.0*	11.52 *63.2*
2	2	1	10.00 *100.0*	7.50 *75.0*	15.00 *100.0*	17.50 *116.5*	20.00 *100.0*	15.00 *75.0*	16.66 *100.0*	15.0 *90.0*	13.33 *100.0*	20.00 *150.0*
3	2	0.5	10.87 *100.0*	5.00 *46.0*	14.13 *100.0*	20.00 *141.5*	21.70 *100.0*	10.00 46.1	19.54 *100.0*	10.00 *51.2*	8.70 *100.0*	30.00 *345.0*
4	1	2	8.09 *100.0*	9.62 *119.00*	16.91 *100.0*	15.38 *90.9*	16.18 *100.0*	19.24 *119.3*	13.23 *100.0*	19.24 *145.5*	20.60 *100.0*	11.52 *55.9*
5	1	1	9.38 *100.0*	7.50 *79.9*	15.62 *100.0*	17.50 *111.9*	18.75 *100.0*	15.00 *80.0*	16.66 *100.0*	15.00 *90.0*	14.60 *100.0*	20.00 *137.0*
6	1	0.5	10.53 *100.0*	5.00 *47.5*	14.47 *100.0*	20.00 *138.0*	21.04 *100.0*	10.00 *47.5*	19.74 *100.0*	10.00 *50.6*	9.20 *100.0*	30.00 *326.0*
7	0.5	2	7.32 *100.0*	9.62 *131.3*	17.67 *100.0*	15.38 *87.0*	14.66 *100.0*	19.24 *131.0*	12.93 *100.0*	19.24 *149.0*	22.40 *100.0*	11.52 *51.3*
8	0.5	1	8.93 *100.0*	7.50 *83.9*	16.06 *100.0*	17.50 *108.7*	17.84 *100.0*	15.00 *83.9*	16.66 *100.0*	15.00 *90.00*	15.45 *100.0*	20.00 *129.5*
9	0.5	0.5	10.33 *100.0*	5.00 *48.4*	14.68 *100.0*	20.00 *136.1*	20.61 *100.0*	10.00 *48.5*	19.76 *100.0*	10.0 *50.6*	9.60 *100.0*	30.00 *312.0*

TABLE 28.

EFFECT OF THE PROPORTIONS OF A BENT UPON THE ACCURACY OF METHOD I.

ALL GIRDERS PROPORTIONAL TO THE BENDING MOMENTS.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column *A* and girder *a* are equal, and the ratio of the moment of inertia of girder *a* to the moment of inertia of girder *b* equals the ratio of the bending moment in girder *a* to the bending moment in girder b, as determined by method I.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column *A*		Moment at Top and Bottom of Column *B*		Moment at Right End of Girder *a*		Moment at Left End of Girder *a*		Moment at End of Girder *b*	
	Ratio of Story Height to Width of Bay *a*	Ratio of Width of Bay *a* to Width of Bay *b*	Slope-Deflection Methed	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I
1	2	2	9.75	9.62	15.25	15.38	19.50	19.24	16.05	19.24	14.45	11.52
			100.0	*98.8*	*100.0*	*100.5*	*100.0*	*98.8*	*100.0*	*120.0*	*100.0*	*79.6*
2	2	1	9.60	7.50	15.40	17.50	19.2	15.00	15.60	15.00	15.20	20.00
			100.0	*78.1*	*100.0*	*113.6*	*100.0*	*78.1*	*100.0*	*96.1*	*100.0*	*131.5*
3	2	0.5	9.45	5.00	15.55	20.00	18.9	10.00	15.10	10.00	16.00	30.00
			100.0	*52.9*	*100.0*	*129.0*	*100.0*	*52.9*	*100.0*	*66.3*	*100.0*	*187.5*
4	1	2	9.05	9.62	15.95	15.38	18.10	19.24	15.70	19.24	16.20	11.52
			100.0	*106.2*	*100.0*	*96.2*	*100.0*	*106.2*	*100.0*	*122.5*	*100.0*	*92.6*
5	1	1	8.85	7.50	16.15	17.50	17.70	15.00	15.20	15.00	17.10	20.00
			100.0	*84.8*	*100.0*	*108.3*	*100.0*	*84.8*	*100.0*	*98.8*	*100.0*	*117.0*
6	1	0.5	8.60	5.00	16.40	20.00	17.20	10.00	14.7	10.00	18.1	30.00
			100.0	*58.0*	*100.0*	*122.0*	*100.0*	*58.1*	*100.0*	*68.0*	*100.0*	*165.8*
7	0.5	2	8.55	9.62	16.45	15.38	17.10	19.24	15.80	19.24	17.1	11.52
			100.0	*112.5*	*100.0*	*93.2*	*100.0*	*112.5*	*100.0*	*122.0*	*100.0*	*67.3*
8	0.5	1	8.30	7.50	16.70	17.50	16.60	15.00	15.20	15.00	18.2	20.00
			100.0	*90.3*	*100.0*	*105.0*	*100.0*	*90.4*	*100.0*	*98.8*	*100.0*	*110.0*
9	0.5	0.5	8.00	5.00	17.00	20.00	16.00	10.00	14.60	10.00	19.4	30.00
			100.0	*62.5*	*100.0*	*117.5*	*100.0*	*62.5*	*100.0*	*68.5*	*100.0*	*154.8*

TABLE 29.

EFFECT OF THE PROPORTIONS OF A BENT UPON THE ACCURACY OF METHOD II.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column *A* and girder *a* are equal, and the ratio of the moment of inertia of girder *a* to the moment of inertia of girder *b* equals the ratio of the bending moment in girder *a* to the bending moment in girder *b*, as determined by method II.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column *A*		Moment at Top and Bottom of Column *B*		Moment at Right End of Girder *a*		Moment at Left End of Girder *a*		Moment at End of Girder *b*	
	Ratio of Story Height to Width of Bay *a*	Ratio of Width of Bay a to Width of Bay *b*	Slope-Deflection Method	Method II	Slope-Deflection Method	Method II	Slope-Deflection Method	Method II	Slope-Deflection Method	Method II	Slope-Deflection Method	Method II
1	2	2	9.65 *100.0*	12.50 *129.5*	15.35 *100.0*	12.50 *81.5*	19.30 *100.0*	25.00 *129.5*	15.45 *100.0*	8.33 *53.9*	15.25 *100.0*	16.66 *109.3*
2	2	1	10.55 *100.0*	12.50 *118.5*	14.45 *100.0*	12.50 *86.5*	21.10 *100.0*	25.00 *118.5*	18.30 *100.0*	8.33 *45.6*	10.60 *100.0*	16.66 *157.1*
3	2	0.5	11.25 *100.0*	12.50 *111.0*	13.75 *100.0*	12.50 *91.0*	22.50 *100.0*	25.00 *111.1*	20.95 *100.0*	8.33 *39.8*	6.55 *100.0*	16.66 *255.0*
4	1	2	8.82 *100.0*	12.50 *141.8*	16.18 *100.0*	12.50 *77.3*	17.64 *100.0*	25.00 *141.8*	15.26 *100.0*	8.33 *54.6*	17.10 *100.0*	16.66 *97.5*
5	1	1	10.10 *100.0*	12.50 *123.8*	14.90 *100.0*	12.50 *83.9*	20.20 *100.0*	25.00 *124.0*	18.45 *100.0*	8.33 *45.20*	11.35 *100.0*	16.66 *146.8*
6	1	0.5	11.00 *100.0*	12.50 *113.5*	14.00 *100.0*	12.50 *89.3*	22.00 *100.0*	25.00 *113.7*	21.20 *100.0*	8.33 *39.3*	6.80 *100.0*	16.66 *245.0*
7	0.5	2	8.30 *100.0*	12.50 *150.6*	16.70 *100.0*	12.50 *74.9*	16.60 *100.0*	25.00 *150.7*	15.20 *100.0*	8.33 *54.9*	18.20 *100.0*	16.66 *91.6*
8	0.5	1	9.80 *100.0*	12.50 *127.5*	15.20 *100.0*	12.50 *82.3*	19.60 *100.0*	25.00 *127.0*	18.55 *100.0*	8.33 *44.9*	11.85 *100.0*	16.66 *140.5*
9	0.5	0.5	10.85 *100.0*	12.50 *115.2*	14.15 *100.0*	12.50 *88.4*	21.70 *100.0*	25.00 *115.0*	21.40 *100.0*	8.33 *38.9*	6.9 *100.0*	16.66 *241.5*

Table 30.

Effect of the Proportions of a Bent Upon the Accuracy of Method III.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column *A* and girder *a* are equal, and the ratio of the moment of inertia of girder *a* to the moment of inertia of girder *b* equals the ratio of the bending moment in girder *a* to the bending moment in girder *b*, as determined by method III.

For each bent the upper line is the moment in per cent of W x h, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column *A*		Moment at Top and Bottom of Column *B*		Moment at Right End of Girder *a*		Moment at Left End of Girder *a*		Moment at End of Girder *b*	
	Ratio of Story Height to Width of Bay *a*	Ratio of Width of Bay *a* to Width of Bay *b*	Slope-Deflection Method	Method III	Slope-Deflection Method	Method III	Slope-Deflection Method	Method III	Slope-Deflection Method	Method III	Slope-Deflection Method	Method III
1	2	2	10.12 *100.0*	12.50 *123.5*	14.88 *100.0*	12.50 *84.1*	20.24 *100.0*	25.00 *123.5*	17.06 *100.0*	13.48 *79.0*	12.70 *100.0*	11.52 *90.8*
2	2	1	10.30 *100.0*	12.50 *121.5*	14.70 *100.0*	12.50 *85.2*	20.60 *100.0*	25.00 *121.5*	17.60 *100.0*	5.0 *28.4*	11.80 *100.0*	20.00 *170.0*
3	2	0.5	10.67 *100.0*	12.50 *117.4*	14.33 *100.0*	12.50 *87.3*	21.34 *100.0*	25.00 *117.3*	18.79 *100.0*	—5.0	9.87 *100.0*	30.00 *304.5*
4	1	2	9.55 *100.0*	12.50 *131.0*	15.45 *100.0*	12.50 *80.9*	19.10 *100.0*	25.00 *131.0*	17.00 *100.0*	13.48 *79.3*	13.90 *100.0*	11.52 *83.0*
5	1	1	9.80 *100.0*	12.50 *127.7*	15.20 *100.0*	12.50 *82.3*	19.60 *100.0*	25.00 *127.6*	17.60 *100.0*	5.0 *28.4*	12.80 *100.0*	20.00 *156.3*
6	1	0.5	10.26 *100.0*	12.50 *122.0*	14.74 *100.0*	12.50 *84.9*	20.52 *100.0*	25.00 *122.0*	18.98 *100.0*	—5.0	10.50 *100.0*	30.00 *286.1*
7	0.5	2	9.15 *100.0*	12.50 *136.9*	15.85 *100.0*	12.50 *78.8*	18.30 *100.0*	25.00 *136.8*	17.10 *100.0*	13.48 *78.8*	14.60 *100.0*	11.52 *79.0*
8	0.5	1	9.46 *100.0*	12.50 *132.2*	15.54 *100.0*	12.50 *80.5*	18.92 *100.0*	25.00 *132.1*	17.63 *100.0*	5.0 *28.4*	13.45 *100.0*	20.00 *148.8*
9	0.5	0.5	10.05 *100.0*	12.50 *124.5*	14.95 *100.0*	12.50 *83.70*	20.10 *100.0*	25.00 *124.5*	18.95 *100.0*	—5.0	10.95 *100.0*	30.00 *274.5*

TABLE 31.

EFFECT OF THE PROPORTIONS OF A BENT UPON THE ACCURACY OF METHOD IV.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column A and girder a are equal, and the ratio of the moment of inertia of girder a to the moment of inertia of girder b equals the ratio of the bending moment in girder a to the bending moment in girder b, as determined by method IV.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column A		Moment at Top and Bottom of Column B		Moment at Right End of Girder a		Moment at Left End of Girder a		Moment at End of Girder b	
	Ratio of Story Height to Width of Bay a	Ratio of Width of Bay a to Width of Bay b	Slope-Deflection Method	Method IV	Slope-Deflection Method	Method IV	Slope-Deflection Method	Method IV	Slope-Deflection Method	Method IV	Slope-Deflection Method	Method IV
1	2	2	9.09 *100.0*	8.33 *91.8*	15.91 *100.0*	16.66 *104.5*	18.18 *100.0*	16.66 *91.7*	13.62 *100.0*	16.66 *122.1*	18.20 *100.0*	16.66 *91.5*
2	2	1	10.0 *100.0*	8.33 *83.3*	15.00 *100.0*	16.66 *111.1*	20.00 *100.0*	16.66 *83.3*	16.66 *100.0*	16.66 *100.0*	13.33 *100.0*	16.66 *125.0*
3	2	0.5	10.89 *100.0*	8.33 *76.8*	14.13 *100.0*	16.66 *117.8*	21.74 *100.0*	16.66 *76.6*	19.56 *100.0*	16.66 *85.20*	8.70 *100.0*	16.66 *191.5*
4	1	2	8.09 *100.0*	8.33 *103.2*	16.91 *100.0*	16.66 *98.3*	16.18 *100.0*	16.66 *103.0*	13.22 *100.0*	16.66 *126.0*	20.60 *100.0*	16.66 *80.9*
5	1	1	9.38 *100.0*	8.33 *88.8*	15.62 *100.0*	16.66 *106.4*	18.76 *100.0*	16.66 *88.9*	16.64 *100.0*	16.66 *100.0*	14.60 *100.0*	16.66 *114.0*
6	1	0.5	10.53 *100.0*	8.33 *79.2*	14.47 *100.0*	16.66 *115.0*	21.06 *100.0*	16.66 *79.1*	19.74 *100.0*	16.66 *84.4*	9.20 *100.0*	16.66 *181.0*
7	0.5	2	7.32 *100.0*	8.33 *113.9*	17.68 *100.0*	16.66 *94.2*	14.64 *100.0*	16.66 *113.8*	12.96 *100.0*	16.66 *128.7*	22.40 *100.0*	16.66 *74.4*
8	0.5	1	8.93 *100.0*	8.33 *93.3*	16.07 *100.0*	16.66 *103.7*	17.86 *100.0*	16.66 *93.3*	16.69 *100.0*	16.66 *100.0*	15.45 *100.0*	16.66 *107.8*
9	0.5	0.5	10.33 *100.0*	8.33 *80.7*	14.67 *100.0*	16.66 *113.5*	20.66 *100.0*	16.66 *80.6*	19.74 *100.0*	16.66 *84.4*	9.60 *100.0*	16.66 *173.5*

TABLE 32.

LOG OF THE TEST OF CELLULOID MODEL NO. 4.

Story No.	Shear W in Lb.	Horizontal Deflection at Center of Girder at Top of Story in Inches			Change in Slope Measured on an Arc Having a 19-inch Radius, in Inches.							
					Force at Top of Model Acting Toward the Right				Force at Top of Model Acting Toward the Left			
					θ at Top of Column A		θ at Top of Column B		θ at Top of Column A		θ at Top of Column A	
		Right-Hand Span	Mid-dle Span	Left-Hand Span	Right-Hand Side of Model	Left-Hand Side of Model	Right-Hand Side of Model	Left-Hand Side of Model	Right-Hand Side of Model	Left-Hand Side of Model	Right-Hand Side of Model	Left-Hand Side of Model
1	1.5	.04	.04	.04	.20	.20	.09	.10	.19	.18	.09	.10
	3.0	.08	.08	.08	.40	.40	.20	.21	.40	.40	.19	.23
	4.5	.11	.11	.11	.61	.59	.31	.33	.61	.63	.30	.38
2	1.5	.08	.08	.08	.21	.21	.10	.10	.19	.19	.11	.10
	3.0	.16	.16	.16	.43	.43	.21	.20	.41	.41	.24	.22
	4.5	.25	.24	.24	.66	.66	.34	.33	.63	.65	.37	.37
3	1.5	.14	.14	.14	.18	.17	.11	.12	.17	.16	.13	.14
	3.0	.28	.29	.28	.37	.35	.26	.26	.36	.35	.26	.29
	4.5	.42	.42	.42	.59	.54	.39	.40	.56	.55	.40	.42
4	1.5	.18	.18	.18	.13	.10	.06	.07	.13	.09	.06	.07
	3.0	.36	.36	.37	.25	.19	.13	.15	.26	.20	.13	.15
	4.5	.55	.55	.55	.40	.30	.20	.22	.39	.27	.18	.25

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